Learning Mathematics Using TI-84 PLUS Graphing Calculator

CURRICULUM DEVELOPMENT DIVISION
MINISTRY OF EDUCATION MALAYSIA

Learning Mathematics

Using TI-84 Plus Graphing Calculator

First Printing 2010

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Curriculum Development Division
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Contents

PREFACE

Education must change to keep pace with the demands of the present world. Learning in the 21st century must leverage the affordances of new technologies, employ better pedagogies based on recent research on student learning, and be cognizant of the characteristics of a new breed of learners.

The Ministry of Education (MOE) acknowledges that it is vital to prepare pupils for life in today's highly technical society. Pupils' mathematical knowledge must be included to go beyond the simple skills into solving more complex problems. With this realisation, the MOE continuously advocates integration of technology in the teaching and learning of mathematics to develop the intellectual capital and educate students to think creatively.

Effective use of technology however, requires teachers to introduce changes in teaching strategies and move away from teacher- to pupil-centred activities. Teachers need to take up the role as facilitators and pier guides; and teaching and learning has to change from memorisation and rote learning culture to the culture of problem solving and generation of knowledge.

Graphing calculator is seen as an essential tool for doing and learning mathematics in the classroom. This technology is believed to permit students to focus on mathematical ideas, to reason, and solve problems in ways that are often difficult or impossible by traditional means. The graphing calculator enhances the learning of mathematics by allowing for increased exploration and enhanced representation of ideas. Range of problems that can be assessed is also extended.

Having invested substantially in technologies, the MOE need to ensure that teachers are trained and support materials provided to enable them to integrate technology effectively in their teaching and learning. The Learning Mathematics using Graphing Calculator Series is thus initiated to help in the development of pupils' mathematical thinking skills, enhancing the mathematical discourse when pupils investigate and interact with each other and teachers as well. This initiative is based on the belief that technical-graphical-based tools provide better experience for pupils in the learning of mathematics. The content of this particular module is based on TI-84 Plus Graphing Calculator key-stroke which will be improved and added on to from time to time. Modules based on different graphing calculator will follow.

Last but not least, MOE would like to express much gratitude and appreciation to the teachers and MOE officers who contributed to the development of this module in the Learning Mathematics using Graphing Calculator Series.

(DATU Dr HJ. JULAIHI HJ. BUJANG)
Director
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ACKNOWLEDGEMENT

The Curriculum Development Division, Ministry of Education wishes to express our deepest gratitude and appreciation to all panels of contributors for their expert views, opinions, dedications and continuous support in development of this module

Introduction

Background

This module series is especially targeted at pupils taking the Mathematics and Additional Mathematics at the upper secondary level. This one of the continuous efforts initiated by Curriculum Development Division, Ministry of Education, to ensure the teaching and learning can be done more interactively and effectively using graphing calculators. The MOE believes that the use of graphing calculators will help pupils visualise concepts as they make connections with data. When pupils can actually see expressions, formulas, graphs, and the result of changing a variable on those visual representations, a deeper understanding of concept can result.

TI-83 Plus, TI-84 Plus Graphic Calculator

The TI-83 Plus or TI-84 Plus graphing calculator, models from Texas-Instrument, is a hand-held tool that can be easily learnt by teachers, students, and those interested. All commands are placed in neatly arranged pull-down menus, and in the event that one cannot find the commands, the calculator's catalogue gives an alphabetically arranged list of all the commands. The keys have also been strategically arranged in functional groups for easy access to the user.

The models can handle real and complex numbers, matrices and even strings. Its features for trigonometry, calculus, and simple algebra in the form of an equation solver will also meets the needs of most secondary school curriculum. In addition, it can carry out list-based one- and two-variable statistical analysis. The descriptive statistics and linear regression models are applicable to Malaysian secondary school curriculum, whilst the calculator's advanced hypothesis testing, confidence intervals and distributions.

Objective

The objective of this module is to suggest some activities that can be carried out by teachers using Graphing Calculator from Texas-Instrument model, particularly TI-83 Plus or TI-84 Plus during their respective lessons. This module consist both elements in Mathematics and Additional Mathematics. It focuses on upper-form syllabus which includes all components such as Algebraic, Geometric, Statistics, Trigonometric and Calculus. It intends to enable students to investigate and apply mathematical ideas in a way not easily achieved by conventional means.

Module layout

This module encompasses some of topics in form 4 and form 5. It comprising subjects in Mathematics and Additional Mathematics as follows:-

No	Subjects	Topics	
		Form 4	Form 5
1	Mathematics	• The Straight Line • Statistics • Trigonometry 2	• Graph of Functions 2 • Matrix • Gradient and Area under the graph • Probability 2
2	Additional Mathematics	• Functions • Quadratic Functions • Simultaneous Equation • Coordinate Geometry • Differentiation	• Progression • Linear Law • Integration • Trigonometric functions • Linear Programming

The principle layout for the activities may include:-

- ***Topic***
- ***Lesson Objective***
- ***Table of Procedure, Screenshot/Key stroke, and notes***
- ***Investigations***
- ***Teachers' guide***
- ***Students' Worksheet***
- ***Enrichment***

TOPIC : THE STRAIGHT LINE

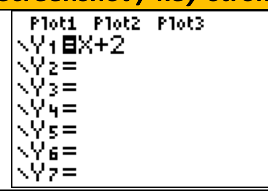
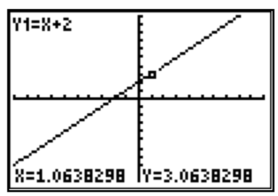
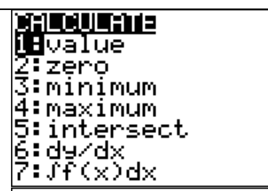
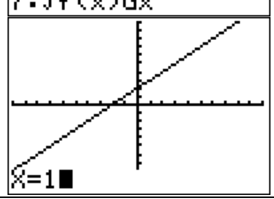
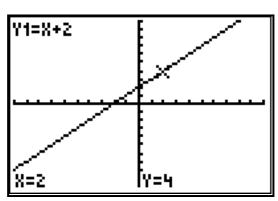
LESSON OBJECTIVE :

Students will be able to…

i. *Draw the graph given an equation of the form $y = mx + c$.*
ii. *Determine whether a given point lies on a specific straight line.*
iii. *Verify that m is the gradient and c is the y-intercept of a straight line with equation $y = mx + c$.*
iv. *Explore properties of parallel lines.*

EXAMPLE QUESTION : *Draw the graph, $y = x + 2$, and answer the questions that follow:*

a) Find the value of y given the $x = 1$
b) Determine whether these points lies on the graph drawn:-
i) $(2,4)$
ii) $(-1,2)$

Step	Procedure	Screenshot / key-stroke	Notes
1	*Draw the graph of $y = x + 2$.* *Then, press GRAPH.*		*Remember to reset the RAM.*
2	*Press TRACE to read the value of y if x=1.*		*You will find that your answer is in the form of decimals and you'll find that it's very hard to bring the cursor **exactly at x = 1.***
3	*Press 2nd TRACE for [CALC], then choose **[1:value]**, and press ENTER.* *Then, key-in **x = 1**,* *press ENTER.*		*(Question a)* *Find the value of y given the $x = 1$* *Then you will get* *y = ………………*
4	*Press 2nd TRACE for [CALC], then choose **[1:value]*** *press ENTER.* ***Then, key-in the x-value from every coordinate. See the y-value for comparison and to have the answer.***		*(Question b)* *Determine whether these points lie on the straight line $y = x + 2$.* *a) (2 , 4) [Answer:…………..]* *b) (-1 , 2) [Answer:…………]*

DISCUSSION

1. *By using the same method, find the value of* y *when* $x = -5$ *(SPM formatted question).*

   $y = -3$

2. *How do you determine whether the point (-1, 2) satisfy the equation* $y = x + 2$*?*

   By substituting the value of x *and* y *in the equation*

3. *Are these points lies on the straight line given below?*

Points / *functions*	*(6,3)*	*(5,-7)*	*(2,4)*
1. $y = x - 3$	***Yes***	*No*	*No*
2. $y = 3x - 2$	*No*	*No*	***Yes***
3. $2y + 4x = 6$	*No*	***Yes***	*No*

STUDENT'S WORKSHEET

DISCUSSION

1. *By using the same method, find the value of* y *when* $x = -5$ *(SPM formatted question).*

   ………

2. *How do you determine whether the point (-1, 2) satisfy the equation* $y = x + 2$*?*

   ………

3. *Are these points lies on the straight line given below?*

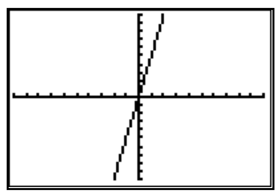
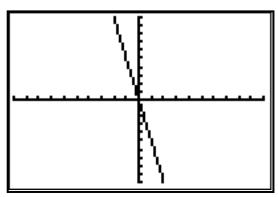
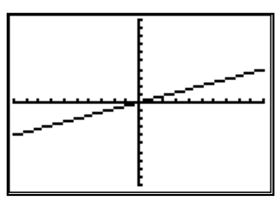
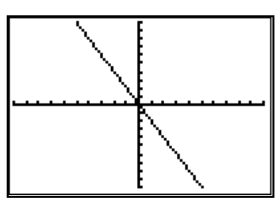
Points / *functions*	*(6,3)*	*(5,-7)*	*(2,4)*
1. $y = x - 3$			
2. $y = 3x - 2$			
3. $2y + 4x = 6$			

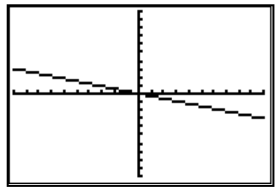
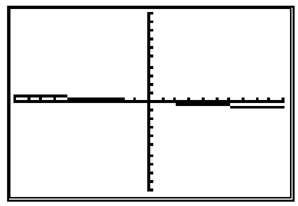
TEACHER'S NOTE

ACTIVITY 1

Complete the table below with appropriate value of the gradient, m the x -intercept, and the y -intercept using graphing calculator, given the function $\boldsymbol{y = mx + c}$

a) Press [Y=]
b) Key in the linear function given. Example, $\boldsymbol{y = x + 5}$
c) Press [X,T,Θ,n] [+] [5] [GRAPH]

No	***Function***	***Value of m***	***Sketch your graph***	***x-intercept***	***y-intercept***
1.	$y = 5x$	5		0	0
2.	$y = -5x$	-5		0	0
3.	$y = 0.4x$	0.4		0	0
4.	$y = \ -2x$	-2		0	0

No	Function	Value of m	Sketch your graph	x-intercept	y-intercept
5.	$y = -0.3x$	-0.3		0	0
6.	$y = 0.08x$	0.08		0	0

DISCUSSION

1. *Compare your answers with your friends, and present your answers in the class.*

   a) *Compare graph no. 1, 3 and 6 with the graph $y = x$.*

   Is it steeper or less steep than the graph $y = x$?

   Results: (Fill in the blanks with steeper or less steep)

   $y = 5x$ *is steeper than* $y = x$.

   $y = 0.4x$ *is less steep than* $y = x$.

   $y = 0.08x$ *is less steep than* $y = x$.

   b) *Compare graph no. 2, 4 and 5 with the graph $y = -x$.*

   Is it steeper or less steep than the graph $y = -x$?

   Results: (Fill in the blanks with steeper or less steep)

   $y = -5x$ *is steeper than* $y = -x$.

   $y = -2x$ *is steeper than* $y = -x$.

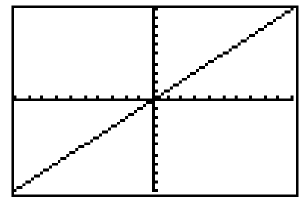
   $y = -0.3x$ *is less steep than* $y = -x$.

c) *Are there any difference between the shape of graph (no. 1, 3 and 6) with the graph (no. 2, 4 and 5)? Why?*

Difference in inclination

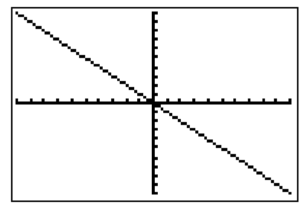
d) *Does the value of m affect the x-intercept or the y-intercept?*

No



e) *If the graph lies in quadrant I and III, what is the sign for m value? (Negative or positive)* *(Positive)*

f) *If the graph lies in quadrant II and IV, what is the sign for m value? (Negative or positive)* *(Negative)*



g) *Let say m is positive. What will happen to the graph as the value of m gets larger?*

The line will approach y-axis, or it will be steeper

h) *What will happen to the graph if the value of $m = 0$?*

The line will be parallel to x-axis, called horizontal line

Verify your answer with the graphing calculator.

CONCLUSION

Make a conclusion for the role of $\boldsymbol{m}$ in the graph $\boldsymbol{y} = \boldsymbol{mx}$.

i. ***If the value of m increase, then the steepness of the line graph increase***
ii. ***If the value of m is positive, the graph lies on quadrant I and III***
iii. ***If the value of m is negative, the graph lies on quadrant II and IV***

STUDENT'S WORKSHEET

ACTIVITY 1

Complete the table below with appropriate value of the gradient, m the x -intercept, and the y -intercept using graphing calculator, given the function $\boldsymbol{y = mx + c}$

a) Press $\boxed{\text{Y=}}$
b) Key in the linear function given. Example, $\boldsymbol{y = x + 5}$
c) Press $\boxed{\text{X,T,}\Theta,n}$ $\boxed{+}$ $\boxed{5}$ $\boxed{\text{GRAPH}}$

No	Function	Value of m	Sketch your graph	x-intercept	y-intercept
1.	$y = 5x$				
2.	$y = -5x$				
3.	$y = 0.4x$				
4.	$y = -2x$				
5.	$y = -0.3x$				
6.	$y = 0.08x$				

STUDENT'S WORKSHEET

DISCUSSION

1. *Compare your answers with your friends, and present your answers in the class.*

   a) *Compare graph no. 1, 3 and 6 with the graph $y = x$. Is it steeper or less steep than the graph $y = x$?*

      Results: (Fill in the blanks with steeper or less steep)

      $y = 5x$ *is ……………………………… than* $y = x$.
      $y = 0.4x$ *is ……………………………than* $y = x$.
      $y = 0.08x$ *is…………………………… than* $y = x$.

   b) *Compare graph no. 2, 4 and 5 with the graph $y = -x$. Is it steeper or less steep than the graph $y = -x$?*

      Results: (Fill in the blanks with steeper or less steep)

      $y = -5x$ *is ………………………than* $y = -x$.
      $y = -2x$ *is ……………………….. than* $y = -x$.
      $y = -0.3x$ *is……………………….. than* $y = -x$.

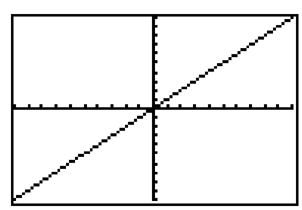
   c) *Are there any difference between the shape of graph (no. 1, 3 and 6) with the graph (no. 2, 4 and 5)? Why?*

      ……

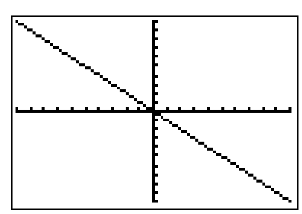
   d) *Does the value of m affect the x-intercept or the y-intercept?*

      ……

   e) *If the graph lies in quadrant I and III, what is the sign for m value? (Negative or positive)……………………………………………………*



   f) *If the graph lies in quadrant II and IV, what is the sign for m value? (Negative or positive)…………………………………………………………….*



g) *Let say m is positive. What will happen to the graph as the value of m gets larger?*

……………………………………………………………………………………………

h) *What will happen to the graph if $m = 0$?*

……………………………………………………………………………………………

Verify your answer with the graphing calculator.

CONCLUSION

Make a conclusion for the role of $\mathbf{m}$ in the graph $y = mx$.

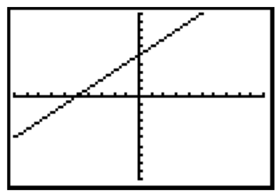
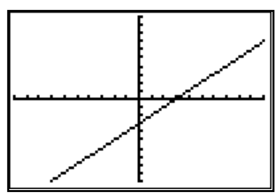
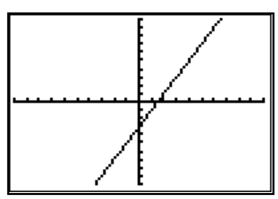
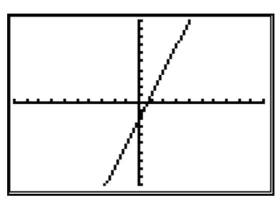
i. ***If the value of $\mathbf{m}$ ……………….. , then the steepness of the line graph ………………….***
ii. ***If the value of $\mathbf{m}$ is …………………, the graph lies on quadrant ………. and …………….***
iii. ***If the value of $\mathbf{m}$ is …………………, the graph lies on quadrant ………. and …………….***

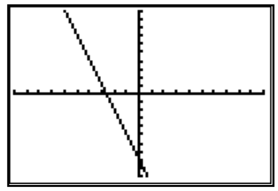
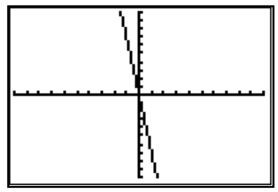
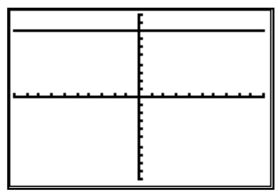
TEACHER'S NOTE

ACTIVITY 2:

Complete the table below with appropriate value of ***the gradient, m, the value c***, ***the x -intercept, and the y -intercept*** *using graphing calculator for the function* $\boldsymbol{y = mx + c}$

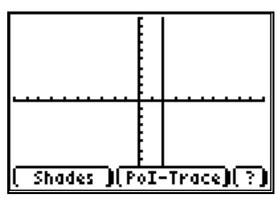
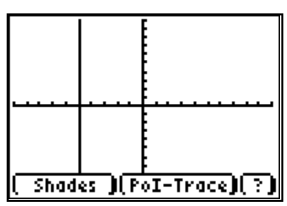
a) Press $\boxed{\text{Y=}}$
b) Key in the linear function given. Example, $\boldsymbol{y = x + 5}$
c) Press $\boxed{\text{X,T,}\Theta\text{,}n}$ $\boxed{+}$ $\boxed{5}$ $\boxed{\text{GRAPH}}$

No	***Function***	***Value of m***	***Value of c***	***Sketch your graph***	***x -intercept***	***y -intercept***
1.	$y = x + 5$	*1*	*5*		-5	*5*
2.	$y = x - 3$	*1*	-3		3	-3
3.	$y = 2x - 3$	*2*	-3		$\frac{3}{2}$	-3
4.	$y = 3x - 2$	*3*	-2		$\frac{2}{3}$	-2

No	Function	Value of m	Value of c	Sketch your graph	x -intercept	y -intercept
5.	$y = -3x - 8$	-3	-8		$\frac{-8}{3}$	-8
6.	$y = -7x$	-7	0		0	0
7.	$y = 8$	none	8		none	8

SELF EXPLORATION

(use $\boxed{APPS}$ Inequalz)

No	Function	Value of m	Value of c	Sketch your graph	x -intercept	y -intercept
8.	$x = 2$	undefined	none		2	none
9	$x = -5$	undefined	none		-5	none

TEACHER'S NOTE

DISCUSSION

1. *Check the answer for each graph. (Compare and contrast)*
2. *Present your answers in the class. (Mathematical Communication)*

   a) *From the table above, try to answer the questions below:*

      i. *For the positive value of c, is the y-intercept above or below the x-axis?*

         above

      ii. *For the negative value of c, is the y-intercept above or below the x-axis?*

         below

      iii. *What is the y-intercept for the equation* $y = 5x + 2$?

         2

      iv. *What is the y-intercept for the equation* $y = mx + c$?

         c value

   b) *Use the graphing calculator to draw the graphs* $y = 3x + 4$ *and* $y = 3x - 4$. *Describe and compare both graphs*

      They are parallel or the gradients are the same but the y-intercept is different

   c) *How does the graph* $y = 0.5$, $y = 8$ *and* $y = 11$ *looks like? What are their common characteristics?*

      They are all horizontal lines

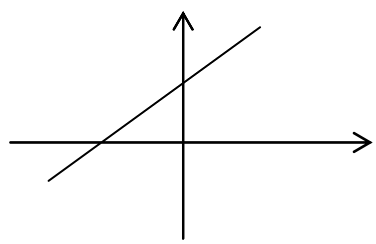
   d) *How does the graph* $x = 0.5$, $x = 8$ *and* $x = 11$ *looks like? What are their common characteristics?*

      They are all vertical lines

3. Guess the equation for the graphs below for $y = mx + c$. Label the axis, and the y-intercept.

<table>
<tr>
<td>

a)



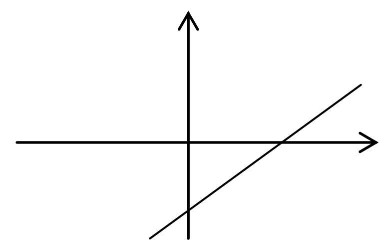
Equation: ………………………………………

Value of m *?* ……………………

Value of c *?* ………………………

</td>
<td>

b)



Equation: ………………………………………

Value of m *?* ……………………

Value of c *?* ………………………

</td>
</tr>
<tr>
<td colspan="2">

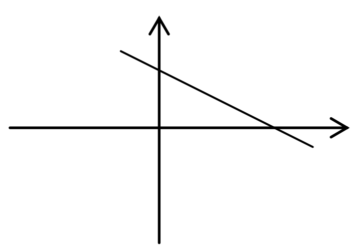
Common rules:

1) The value for m is positive

2) the value of c shows the y-intercept

</td>
</tr>
<tr>
<td>

c)



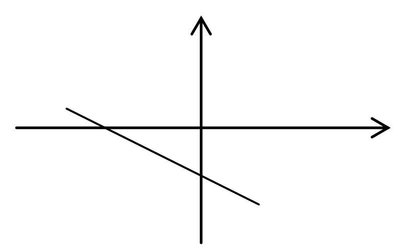
Equation: ………………………………………

Value of m *?* ……………………

Value of c *?* ………………………

</td>
<td>

d)



Equation: ………………………………………

Value of m *?* ……………………

Value of c *?* ………………………

</td>
</tr>
<tr>
<td colspan="2">

Common rules:

1) The value for m is negative

2) the value of c shows the y-intercept

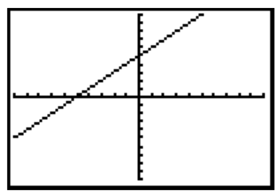
</td>
</tr>
</table>

STUDENT'S WORKSHEET

ACTIVITY 2:

*Complete the table below with appropriate value of **the gradient, m, the value c, the x -intercept, and the y -intercept** using graphing calculator for the function $y = mx + c$*

a) Press $\boxed{\text{Y=}}$
b) Key in the linear function given. Example, $y = x + 5$
c) Press $\boxed{\text{X,T,}\Theta,n}$ $\boxed{+}$ $\boxed{5}$ $\boxed{\text{GRAPH}}$

No	Function	Value of m	Value of c	Sketch your graph	x -intercept	y -intercept
1.	$y = x + 5$					
2.	$y = x - 3$					
3.	$y = 2x - 3$					
4.	$y = 3x - 2$					

No	Function	Value of m	Value of c	Sketch your graph	x -intercept	y -intercept
5.	$y = -3x - 8$					
6.	$y = -7x$					
7.	$y = 8$					

SELF EXPLORATION

(use $\boxed{APPS}$ Inequalz)

No	Function	Value of m	Value of c	Sketch your graph	x -intercept	y -intercept
8.	$x = 2$					
9	$x = -5$					

STUDENT'S WORKSHEET

DISCUSSION

1. *Check the answer for each graph. (Compare and contrast)*
2. *Present your answers in the class. (Mathematical Communication)*

   a) *From the table above, try to answer the questions below:*

      i. *For the positive value of c, is the y-intercept above or below the x-axis?*

         ……………………………………………………

      ii. *For the negative value of c, is the y-intercept above or below the x-axis?*

         ……………………………………………………

      iii. *What is the y-intercept for the equation $y = 5x + 2$?*

         ……………………………………………………

      iv. *What is the y-intercept for the equation y = mx + c ?*

         ……………………………………………………

   b) *Use the graphing calculator to draw the graphs $y = 3x + 4$ and $y = 3x - 4$. Describe and compare both graphs*

      ……

   c) *How does the graph y = 0.5, y = 8 and y = 11 looks like? What are their common characteristics?*

      ……

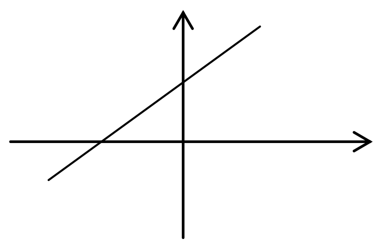
   d) *How does the graph x = 0.5, x = 8 and x = 11 looks like? What are their common characteristics?*

      ……

3. Guess the equation for the graphs below for $y = mx + c$. Label the axis, and the y-intercept.

<table>
<tr><td>

a)



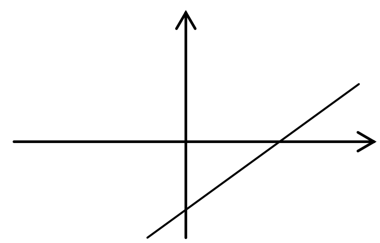
Equation: ……………………………………………

Value of m ? ………………………

Value of c ? ……………………….

</td><td>

b)



Equation: ……………………………………………

Value of m ? ………………………

Value of c ? ……………………….

</td></tr>
<tr><td colspan="2">

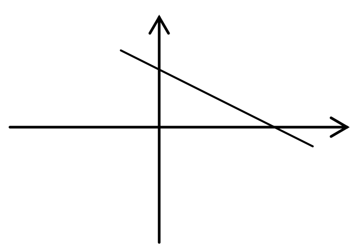
Common rules:

1) ………………………………………………………………………………………

2) ………………………………………………………………………………………

</td></tr>
<tr><td>

c)



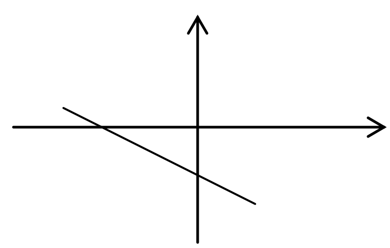
Equation: ……………………………………………

Value of m ? ………………………

Value of c ? ……………………….

</td><td>

d)



Equation: ……………………………………………

Value of m ? ………………………

Value of c ? ……………………….

</td></tr>
<tr><td colspan="2">

Common rules:

1) ………………………………………………………………………………………

2) ………………………………………………………………………………………

</td></tr>
</table>

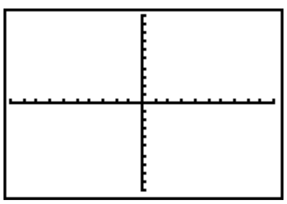
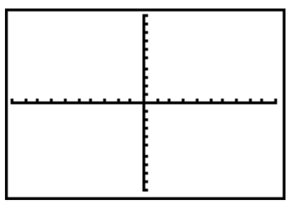
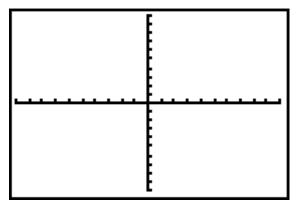
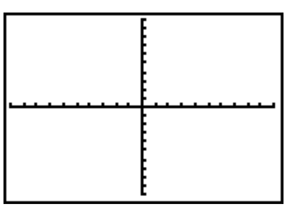
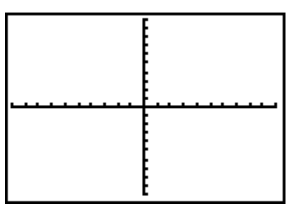
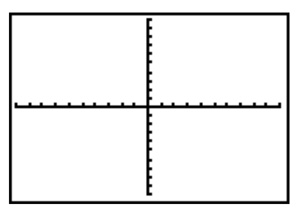
TEACHER'S NOTE

ACTIVITY 3:

1. *Draw a pair of graph simultaneously in each question given in Table 1 with the aid of graphing calculator.*
2. *Write the point of intersection for each question. Press 2nd TRACE* ***for*** *[CALC] mode, choose* ***[5:intersect],*** *then press ENTER three times (the guessing steps will help you to make conjecture), then intersection will appear.*
3. *The answers obtained from the graphing calculator might not be precise. You may use ZOOM and TRACE to get precise answers.*

(Notes: You might want to rewrite the equations to <u>standard form</u>.)

Table 1

a. $y = 2x + 3$ $y = 2x - 3$  *Point of intersection?* *(……….,…………)* *- no intersection*	*b.* $x - y = 5$ $x - y = -10$  *Point of intersection?* *(……….,…………)* *- no intersection*	*c.* $x + y = -1$ $x + 20 = -2y$  *Point of intersection?* *(……18,-19……)*
d. $2y = -4x + 2$ $2y = x - 16$  *Point of intersection?* *(……3.6….,…-6.2………)*	*e.* $2y = -6x + 2$ $4 = y + 3x$  *Point of intersection?* *(……….,…………)* *- no intersection*	*f.* $-6x + 3y + 15 = 0$ $-9x - 5y + 13 = 0$  *Point of intersection?* *(……2….,…-1………)*

TEACHER'S NOTE

DISCUSSION

1. *Which pair of graphs in Table 1 is parallel?*

   …………a, b……………and……e…………

2. *Why do you say so?*

   They have no point of intersection

3. *Find the gradient of graph (e) in table above?*

   $2y = -6x + 2$; *gradient = -3*

   $4 = y + 3x$; *gradient = -3*

4. *Based on the gradient above, what can you say about the gradient of two parallel lines?*

   a) They have the same gradient value

   b) They will not intersect with each other

5. *Without drawing any graph, determine whether $x + 4y = 10$ and $x = y + 1$ are parallel?*

   They are NOT parallel

6. *Why do you say so?*

   They have different gradient value

CONCLUSION

1. ***When the two lines have the same gradient, then they are parallel***

2. ***When two lines are parallel, they have the same gradient***

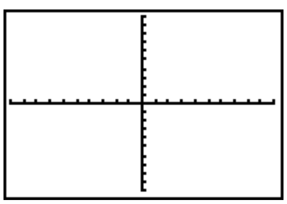
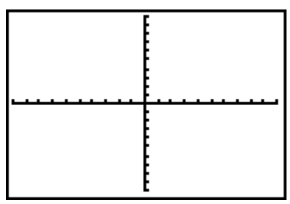
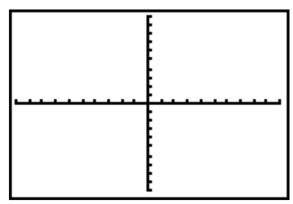
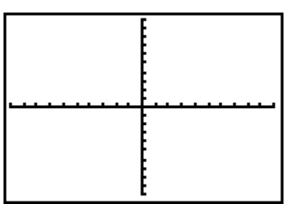
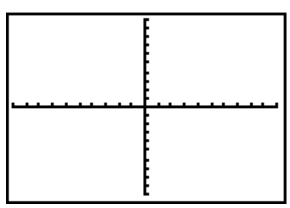
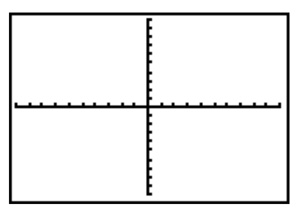
STUDENT'S WORKSHEET

ACTIVITY 3:

1. *Draw a pair of graph simultaneously in each question given in Table 1 with the aid of graphing calculator.*
2. *Write the point of intersection for each question. Press 2nd TRACE **for** [CALC] mode, choose **[5:intersect],** then press ENTER three times (the guessing steps will help you to make conjecture), then intersection will appear.*
3. *The answers obtained from the graphing calculator might not be precise. You may use ZOOM and TRACE to get precise answers.*

(Notes: You might want to rewrite the equations to <u>standard form</u>.)

Table 1

a. $y = 2x + 3$ $y = 2x - 3$  *Point of intersection?* *(……….,…………)*	*b.* $x - y = 5$ $x - y = -10$  *Point of intersection?* *(……….,…………)*	*c.* $x + y = -1$ $x + 20 = -2y$  *Point of intersection?* *(…… , ……)*
d. $2y = -4x + 2$ $2y = x - 16$  *Point of intersection?* *(………. , …………)*	*e.* $2y = -6x + 2$ $4 = y + 3x$  *Point of intersection?* *(………. , …………)*	*f.* $-6x + 3y + 15 = 0$ $-9x - 5y + 13 = 0$  *Point of intersection?* *(………. , …………)*

STUDENT'S WORKSHEET

DISCUSSION

1. *Which pair of graphs in Table 1 is parallel?*

   ………… , …………… and ……………

2. *Why do you say so?*

   ……………………………………………………………………

3. *Find the gradient of graph (e) in table above?*

   $2y = -6x + 2$; *gradient = …………………………………………..*

   $4 = y + 3x$; *gradient = …………………………………………..*

4. *Based on the gradient above, what can you say about the gradient of two parallel lines?*

   a) *They have the same ………………………… value*

   b) *They will …………………………… with each other*

5. *Without drawing any graph, determine whether* $x + 4y = 10$ *and* $x = y + 1$ *are parallel?*

   ……………………………………………………………………

6. *Why do you say so?*

   ……………………………………………………………………

CONCLUSION

1. ***When the two lines have the ……………………. ………., then they are*** *…………………………….*

2. ***When two lines are …………………. , they have the same*** *…………………………….*

TOPIC **:** ***STATISTICS***

LESSON OBJECTIVE **:**

Students will be able to…

i. *Complete the class interval for a set of data given one of the class intervals.*
ii. *Construct a frequency table for a given set of data.*
iii. *Calculate the midpoint of a class.*
iv. *Draw a histogram based on the frequency table of a grouped data.*
v. *Draw the frequency polygon based on histogram or frequency table.*

EXAMPLE QUESTION **:**

The data in Diagram 1 shows the monthly pocket money, in RM, received by 40 students.

32	41	46	56	42	48	51	39
36	47	54	59	34	54	52	48
49	51	62	58	38	63	49	43
56	44	60	64	52	53	55	35
45	38	48	57	44	49	46	40

a) *Based on the data in Diagram 1 and using a class interval of RM5, complete table 1 in the answer space.*

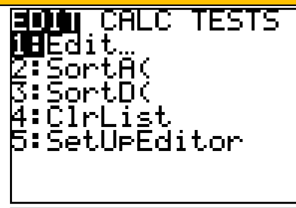
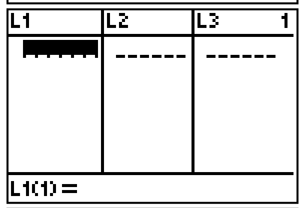
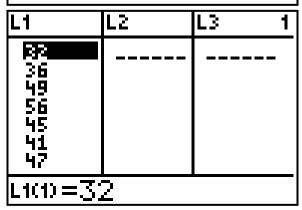
b) *From the table in a)*
   i) *State the modal class*
   ii) *Calculate the mean monthly pocket money of the students.*

c) *By using a scale of 2cm to RM5 on the x-axis and 2cm to 1 student on the y-axis, draw a histogram and frequency polygon based on the data.*

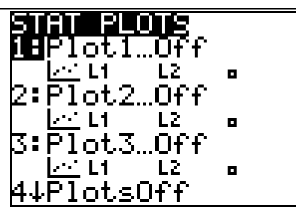
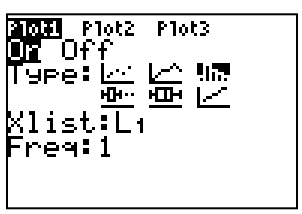
Pocket money(RM)	Frequency	Midpoint
31-35		
36-40		

Table 1

Solution

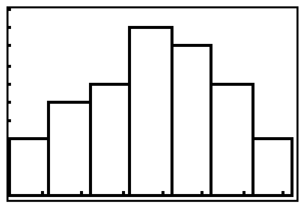
Step	Procedure	Screenshot / key-stroke	Notes
1	*Press* [STAT] *Then, choose **1: Edit**, and press* [ENTER] *Key in **all** the **data** under **L1.***	  	*Reset All RAM before start.* *Press* [2nd] [+] *,* *choose* ***7 : Reset*** ***1: All RAM*** ***2 : Reset*** [ENTER]

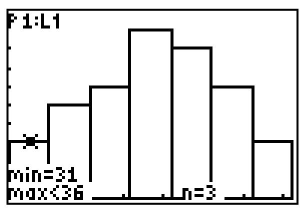
32	41	46	56	42	48	51	39
36	47	54	59	34	54	52	48
49	51	62	58	38	63	49	43
56	44	60	64	52	53	55	35
45	38	48	57	44	49	46	40

Step	Procedure	Screenshot / key-stroke	Notes
2	*Press* [2nd] [Y=] *for* [STAT PLOT] *Then, choose **1: Plot**, and press* [ENTER]		
3	*Under **Plot1**, move cursor and choose **On** by pressing* [ENTER] *Choose Type:* *for histogram.* *Xlist:* ***L1*** *Freq:* ***1***		*Xlist:* ***L1*** *refers to the placement for the list of the data.* *Freq:* ***1*** *refers to the frequency for each data is counted.*

<table>
<tr><td>4</td><td>Press [WINDOW] and key in the following setting</td><td>WINDOW
Xmin=31
Xmax=66
Xscl=5
Ymin=0
Ymax=10
Yscl=1
Xres=1</td><td>• Xmin is the minimum value of the data
• Xmax is the maximum value of the data + 1
• Xscl is the Class Size</td></tr>
<tr><td colspan="4">Questions for discussion

i. Why the maximum value of the data, Xmax must be added 1 value more?

ii. How to determine the class size, Xscl?</td></tr>
<tr><td>5</td><td>Press [GRAPH]</td><td></td><td></td></tr>
<tr><td>6</td><td>Press [TRACE]

Use the right [►] and left [◄] cursor to move from one class interval to another class interval and get the frequency from graph.</td><td>
</td><td>• From [TRACE], the class interval and the frequency can be gained and recorded.
• [TRACE] will show the :
a) Minimum (lower limit) and maximum (upper limit) value of each class, and
b) the frequency of each class
(eg- class 31-35 ,freq = 3)</td></tr>
</table>

Question a)

Based on the data in Diagram 1 and using a class interval of RM5, complete table 1 in the answer space.

Answer:

Pocket money (RM)	Frequency	Mid-point
31-35		
36-40		

Questions for discussion

i. *How to determine the **middle point**?*

ii. *What is the definition of **mod**?*

iii. *How to determine the **modal class**?*

Question b)

From the table in a)

i) State the modal class

Answer : ....

Questions for discussion

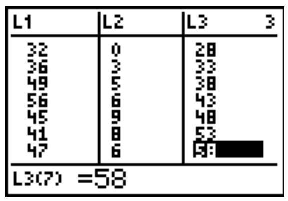
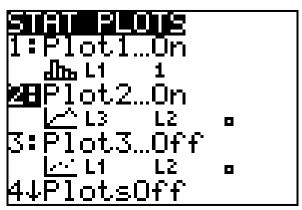
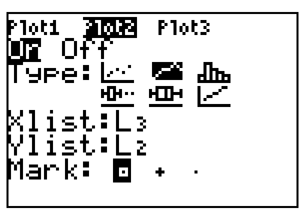
i. *Can you explain the differences between histogram and bar chart?*

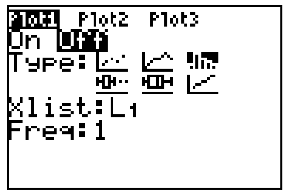
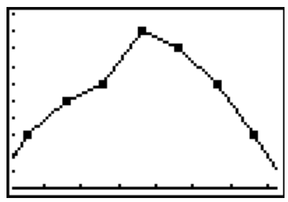
ii. *What is the effect on the histogram if using different class intervals?*

iii. *What must you do to adjust the frequency table for drawing a frequency polygon?*

* ***Additional columns for frequency polygon? Why?***

Pocket money (RM)	***Frequency***	***Mid-point***
*		
31-35		
*		

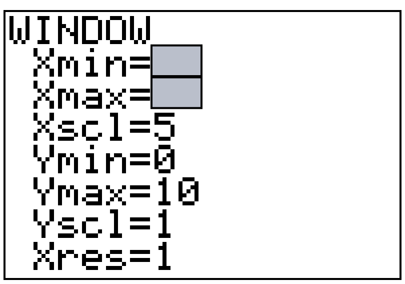
7	*Press [STAT], choose **1: Edit**,* *then press [ENTER]* *Key in the **<u>frequency</u>** in **L2**, **including the additional 0 frequencies*** *Key in the **<u>midpoint</u>** in **L3**, **including the additional class intervals' mid points***		
8	*Press [2nd] [Y=] for [STAT PLOT]* *Choose **2: Plot2**,* *then press [ENTER]* *Move the cursor and choose- **On.*** *Then, press [ENTER]* ***choose type:** for frequency Polygon, and press [ENTER]* *Choose **Xlist: L3**, by pressing [2nd] [3] and **YList: L2**, by pressing [2nd] [2]*	 	

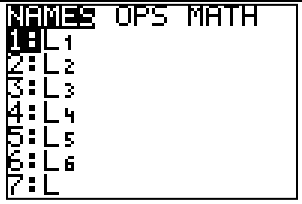
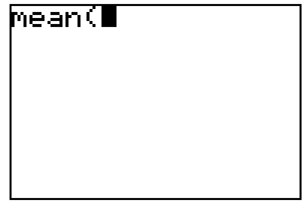
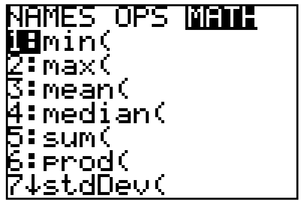
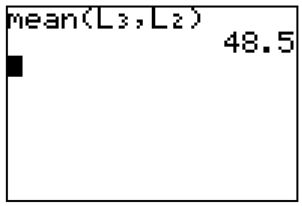
9	*Press* [GRAPH]		
10	*To display* ***ONLY*** *for* ***Frequency Polygon;*** *Press* [2nd] [Y=] *for* [STAT PLOT] *Choose* ***1: Plot1****,* *then, press* [ENTER] *Move the cursor, and choose* ***Off,*** *then press* [ENTER] *Press* [GRAPH]	 	

Questions for discussion

i. *What is wrong with the graph displayed?*

ii. *How should you adjust the display of the window?*



11	*To calculate the **MEAN**,* *Press* [2nd] [STAT]*.* *Move the cursor and choose **MATH***		
	*Choose **mean(** or click for **3***		
	*Key in **mean(L_3,L_2),*** *Then, press* [ENTER]		• *Method for key in the **L_3 and L_2** is by pressing the* [2nd] [3] *and* [2nd] [2]

Question b)

From the table in a)

ii) Calculate the mean monthly pocket money of the students

Answer: …………………………..

TEACHER'S NOTE

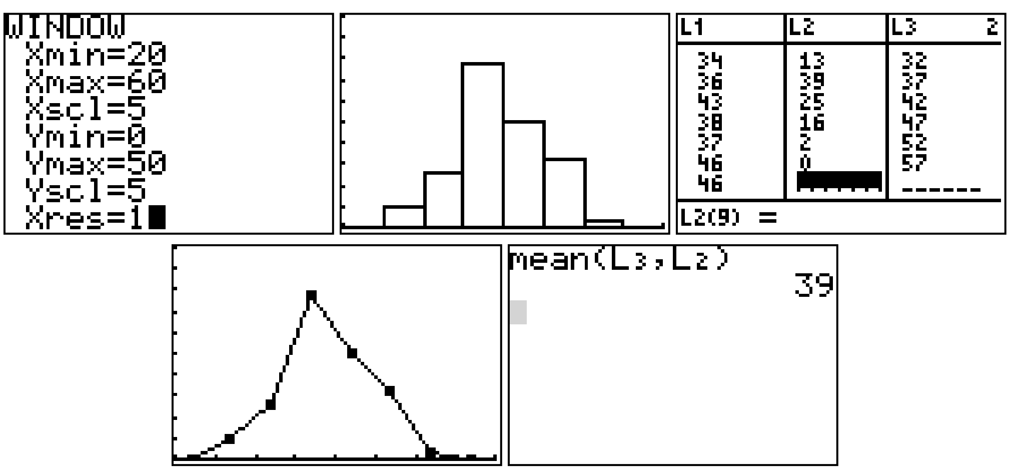
1. *The time taken by 100 students to complete the jogathon in SMK Bestari is recorded in the table below.*

30	38	41	36	26	33	35	38	43	46	36	38	41	36	50	40	47	36	34	37
41	37	31	37	47	41	38	37	35	26	50	37	40	35	45	38	43	40	43	38
34	46	36	40	41	37	33	28	36	30	37	44	39	41	34	38	34	39	33	43
36	46	45	36	33	38	39	32	38	40	29	49	43	36	44	47	38	37	41	47
43	37	35	45	37	41	44	40	46	37	38	45	32	49	40	27	38	47	49	40

a) *Based on the given data, complete the table.*
b) *Construct* ***a histogram and frequency polygon*** *based on the data.*
c) *State the* ***modal class****, and find the* ***mean****.*

SOLUTION:

Time (minutes)	*Frequency*	*Midpoint*
20 – 24	*0*	*22*
25 – 29	*5*	*27*
30 – 34	*13*	*32*
35 – 39	*39*	*37*
40 – 44	*25*	*42*
45 – 49	*16*	*47*
50 – 54	*2*	*52*
55 – 59	*0*	*57*



Modal class = 35 – 39

STUDENT'S WORKSHEET

1. The time taken by 100 students to complete the jogathon in SMK Bestari is recorded in the table below.

30	38	41	36	26	33	35	38	43	46	36	38	41	36	50	40	47	36	34	37
41	37	31	37	47	41	38	37	35	26	50	37	40	35	45	38	43	40	43	38
34	46	36	40	41	37	33	28	36	30	37	44	39	41	34	38	34	39	33	43
36	46	45	36	33	38	39	32	38	40	29	49	43	36	44	47	38	37	41	47
43	37	35	45	37	41	44	40	46	37	38	45	32	49	40	27	38	47	49	40

a) Based on the given data, complete the table.

*b) Construct **a histogram and frequency polygon** based on the data.*

*c) State the **modal class**, and find the **mean**.*

Time (minutes)	Frequency	Midpoint
20 – 24		
25 – 29		

TOPIC : ***STATISTICS***

LESSON OBJECTIVE :

Students will be able to…

i. *Construct the cumulative frequency table for grouped data.*
ii. *Draw the ogive for grouped data.*
iii. *Determine the range of a set of data.*
iv. *Determine the median, the first quartile, the third quartile, and the inter quartile range, from the ogive.*

EXAMPLE QUESTION :

The data in Diagram 1 shows the monthly pocket money, in RM, received by 40 students.

32	41	46	56	42	48	51	39
36	47	54	59	34	54	52	48
49	51	62	58	38	63	49	43
56	44	60	64	52	53	55	35
45	38	48	57	44	49	46	40

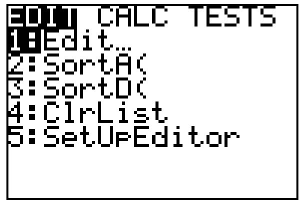
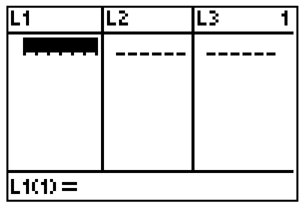
Diagram 1

a) *Based on the data in Diagram 1 and using a class interval of RM5, complete table 2 in the answer space.*
b) *By using a scale of 2cm to RM5 on the x-axis and 2cm to 10 students on the y-axis, draw an ogive.*
c) *From the table in a),*
   i) *Find the median,*
   ii) *Determine the range of the data*
   iii) *Calculate the inter quartile range*

Pocket Money (RM)	*Frequency*	*Upper Boundary*	*Cumulative Frequency*
26-30	0		
31-35	3		

Table 2

Solution

Step	***Procedure***	***Screenshot / key-stroke***	***Notes***
1	*Press* [STAT], *choose* ***1: Edit****,* *then press* [ENTER]	 	

Questions for discussion.

i. *From previous knowledge, can you fill in the class intervals and the frequencies?*

ii. *What is the definition of a boundary?*

iii. *How to determine the upper boundary?*

iv. *What is the definition of the cumulative frequency?*

v. *How to determine the cumulative frequency?*

vi. *What is the definition of range?*

vii. *How to calculate the range?*

Question a)

a) Based on the data in Diagram 1 and using a class interval of RM5, complete table 2 in the answer space.

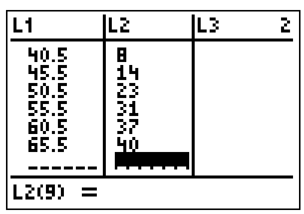
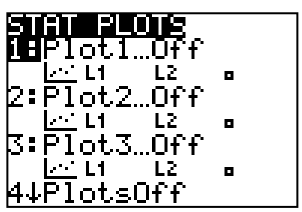
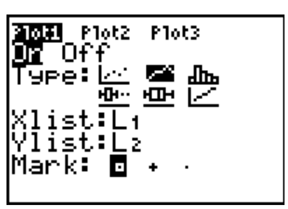
Pocket Money (RM)	*Frequency*	*Upper Boundary*	*Cumulative Frequency*
26-30	*0*		
31-35	*3*		

Question c

ii) Determine the range of the data

range = ☐ - ☐

= ☐

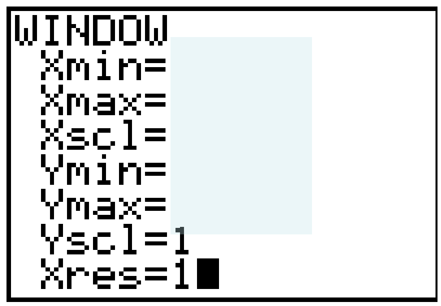
2	*Key in **all** the data from **upper boundary** under **L1** and **cumulative frequency** under **L2.***		
3	*Press [2nd] [Y=] for [STAT PLOT]. Then, choose **1: Plot** and press [ENTER]*		
4	*Under **Plot1**, move the cursor and choose **On*** *Press [ENTER]* *choose type: for ogive.* *Then, press [ENTER]* *Key in for Xlist: **L1*** *Key in for Ylist: **L2***		***Xlist** refers to the data to be plotted with respect to x-axis* ***Ylist** refers to the data to be plotted with respect to y-axis*

<table>
<tr><td>5</td><td colspan="3">

Self exploration

- *Xmin is the* ***minimum value of the data***
- *Xmax is the* ***maximum value of the data + 5***
- *Xscl is the* ***Class Size***
- *Ymax is the* ***total of frequency***

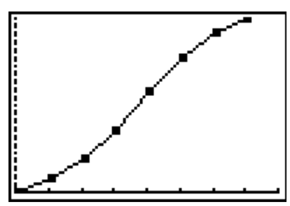
Press [WINDOW] *and key in the appropriate setting*



</td></tr>
<tr><td>5</td><td>

Press [GRAPH]

</td><td>



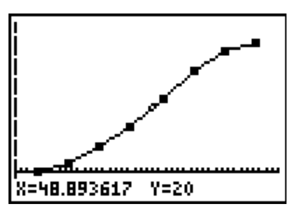
</td><td></td></tr>
<tr><td>6</td><td>

Press [TRACE]

Use the right [▶] *and left* [◀] *to move cursor on the curve to find the median at y = 20.*

Press [ZOOM]*, choose 2:* ***Zoom In*** *to trace y = 20 precisely.*

</td><td>



</td><td>

Question C

i) Find the median

when $y = \frac{1}{2} \times 40$

= 20

Therefore, median is :

Answer: ....

</td></tr>
</table>

Question c (iii)

When $y = \frac{1}{4} \times 40$

$= 10$

Therefore, first quartile is: *Answer: ………………….*

When $y = \frac{3}{4} \times 40$

$= 30$

Therefore, first quartile is: *Answer: ………………….*

Therefore, inter quartile range is = $\boxed{} - \boxed{}$

$= \boxed{}$

TEACHER'S NOTE

2.

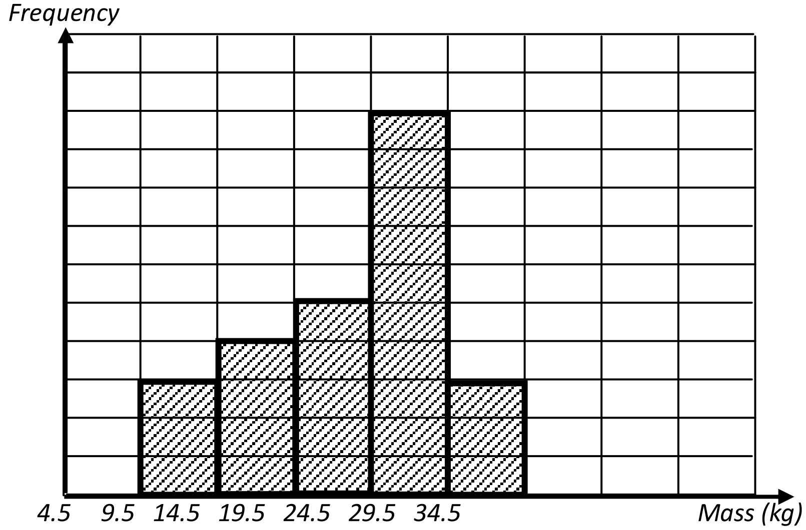


Diagram 2

The histogram in Diagram 2 shows the mass (in kg) for 25 parcels.

(a) *Based on the histogram, complete Table 3.*

(b) *By using a scale of 2 cm to 10 kg on the horizontal axis and 2 cm to 5 parcels on the vertical axis draw an ogive based on Table 3.*

(c) *From your graph, find*

(i) *the median of the mass,*
(ii) *the range of the data*
(ii) *the inter quartile range*

Mass (kg)	Frequency	Upper boundary	Cumulative frequency
5 – 9	0	9.5	0
10 – 14			
15 – 19			
20 – 24			
25 – 29			
30 – 34			

Table 3

SOLUTION:

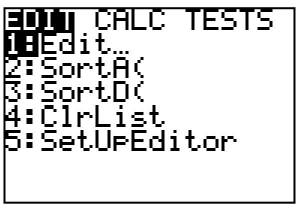
(a)

Mass (kg)	Frequency	Upper boundary	Cumulative frequency
5 – 9	0	9.5	0
10 – 14	3	14.5	3
15 – 19	4	19.5	7
20 – 24	5	24.5	12
25 – 29	10	29.5	22
30 – 34	3	34.5	25

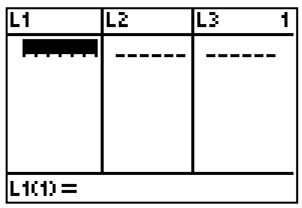
Table 3

Solution for question b) and c), refer steps and procedures in example question

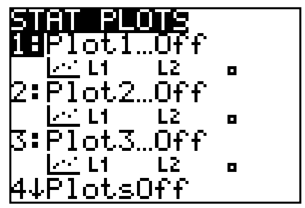
i



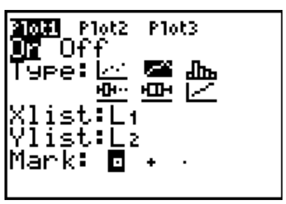
ii



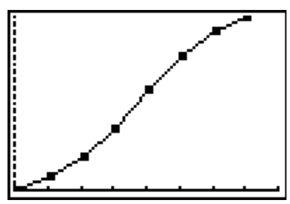
iii



iv



v



STUDENT'S WORKSHEET

2.

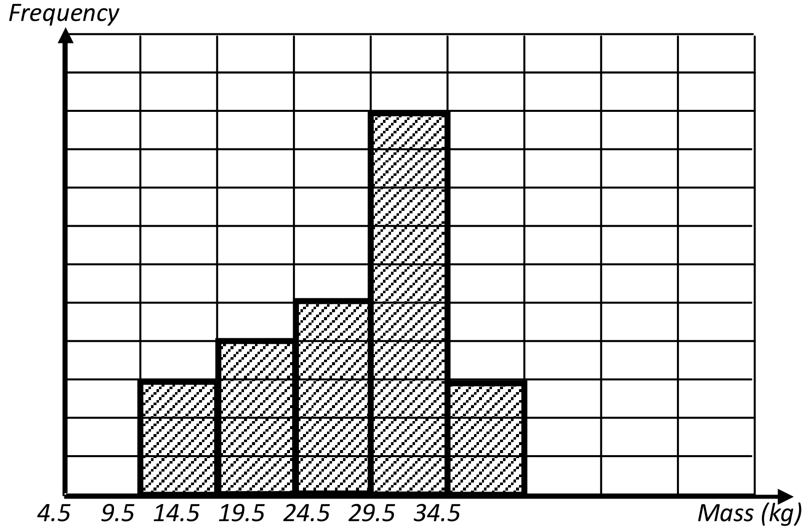


DIAGRAM 2

The histogram in Diagram 2 shows the mass (in kg) for 25 parcels.

(a) Based on the histogram, complete Table 3.

(b) By using a scale of 2 cm to 10 kg on the horizontal axis and 2 cm to 5 parcels on the vertical axis draw an ogive based on Table 3.

(c) From your graph, find

(i) the median of the mass,
(ii) the inter quartile range

Mass (kg)	Frequency	Upper boundary	Cumulative frequency
5 – 9	0	9.5	0
10 – 14			
15 – 19			
20 – 24			
25 – 29			
30 – 34			

Table 3

SOLUTION :

(a)

Mass (kg)	Frequency	Upper boundary	Cumulative frequency
5 – 9	0	9.5	0
10 – 14			
15 – 19			
20 – 24			
25 – 29			
30 – 34			

(b)

(c)

TOPIC **:** ***STATISTICS***

LESSON OBJECTIVES **:**

Students will be able to…

i. Understand and use the concept of class interval
ii. Discuss the effect of the size of class interval on the accuracy of the mean for a specific set of grouped data
iii. Interpret information from a given histogram

EXAMPLE QUESTION **:**

The following table illustrates the number of car accidents that have occurred on a dangerous stretch of a highway over the past 15 years.

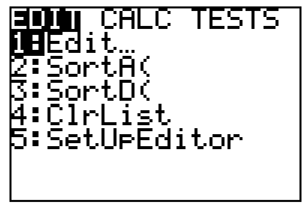
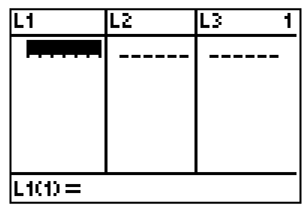
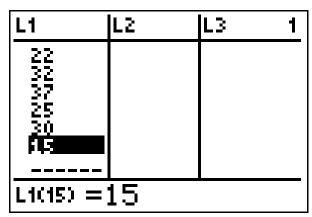
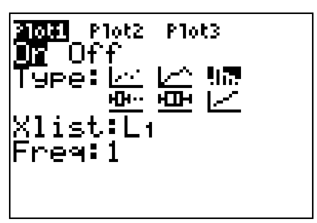
Year	**No. of Accidents**
1992	20
1993	22
1994	21
1995	19
1996	24
1997	27
1998	21
1999	30
2000	31
2001	22
2002	32
2003	37
2004	25
2005	30
2006	15

(a) How can this data be manipulated to suggest that the number of accidents that occur on this stretch highway has increased since 1992? Create a histogram that shows such an increase.

(b) How to alter the horizontal scale of this histogram to create an illustration that suggests that the number of accidents that occur has decreased since 1992 (or remained relatively constant)?

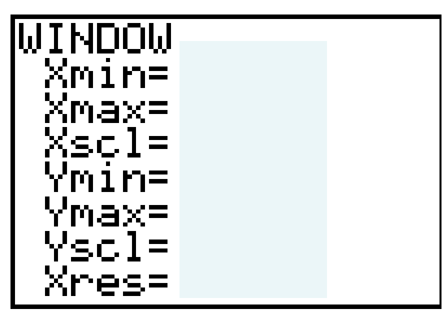
(c) As a driver who frequently travels on this stretch of highway, how do you think that data should be represented such that you see an accurate portrayal of the number of accidents that have occurred?

Solution:

Step	Procedure	Screenshot / key-stroke	Notes
1	*Press* [STAT]*, to enter the STATISTIC mode.* *Choose* ***1: Edit****,* *then press* [ENTER] *Type in the all the data provided as variable in* ***L1****.*	  	
2	Press [2nd][Y=] *under* ***PLOT1****. Select* ***On****. Then, press* [ENTER] Choose **Type:** for Histogram.		

Self exploration

- *Xmin is the* ***minimum value of the data***
- *Xmax is the* ***maximum value of the data + 5***
- *Xscl is the* ***Class Size***
- *Ymax is the* ***total of frequency***



Press [WINDOW] *and key in the appropriate setting*

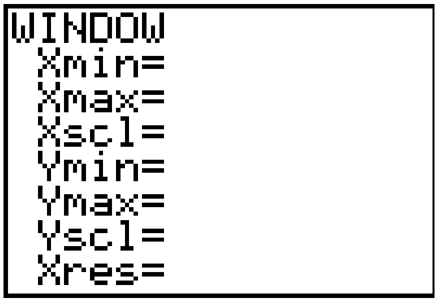
You may change the setting to adjust the look of the histogram.

Press [GRAPH]

Question a)

How can this data be manipulated to suggest that the number of accidents that occur on this stretch highway has ***increased*** *since 1992? Create a histogram that shows such an increase.*

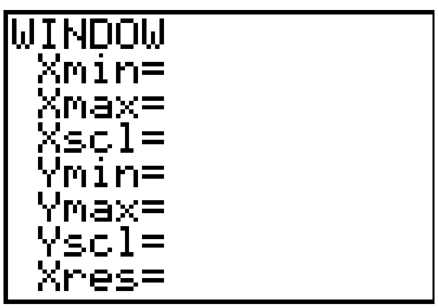
Fill in the **WINDOW** *setting and sketch your histogram.*



Question b)

How to alter the horizontal scale of this histogram to create an illustration that suggests that the number of accidents that occur has ***decreased*** *since 1992 (or remained relatively constant)?*

Fill in the **WINDOW** *setting and sketch your histogram.*



Questions for discussion

i. Explain how you had changed the **WINDOW** *setting such that the graph looks different?*

ii. What interpretation can you tell on histogram in a) and histogram in b)?

iii. Do you think that these two graphs accurately tell you the actual situation considering the number of accidents on this dangerous stretch of highway from 1992 to 2006? Explain.

Question c)

As a driver who frequently travels on this stretch of highway, how do you think that data should be represented such that you see an accurate portrayal of the number of accidents that have occurred?

Questions for discussion

i. How does changing of **Xscl** (or the size of class interval) affect the histogram?

ii. What would you consider before you decide on your value of **Xscl?**

iii. Why does the frequency of the class intervals change when the size of the class interval change?

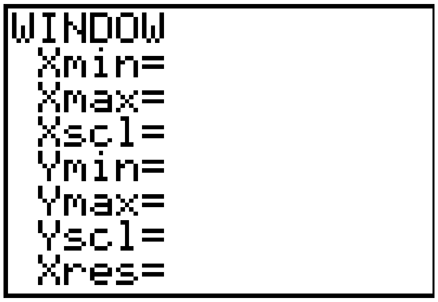
iv. As a driver who frequently travels on this stretch of highway, how do you think the data should be represented such that you see an accurate picture of the number of accidents that have occurred?

v. How do you make histogram represent its data more accurately?

Self exploration

Show the histogram that you will draw to accurately know the actual situation on the number of accidents on that highway.

Fill in the **WINDOW** *setting and sketch your histogram.*



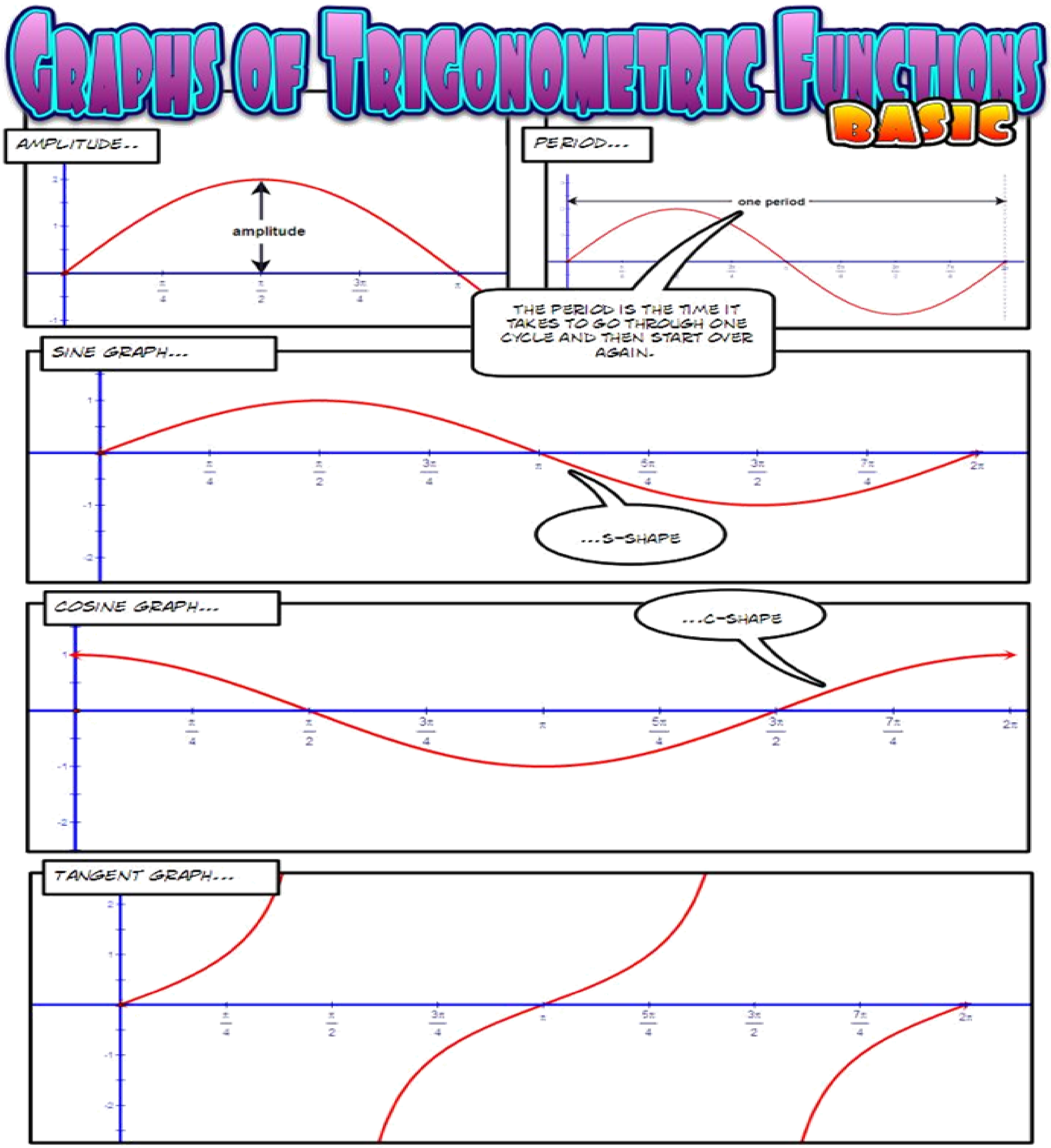
In conclusion, how would you determine a suitable class interval for a given set of data?

TOPIC **:** ***TRIGONOMETRY 2***

LESSON OBJECTIVE **:**

Students will be able to…

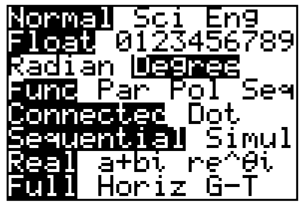
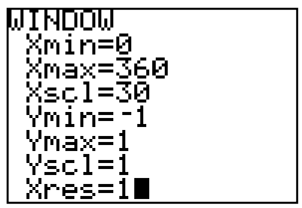
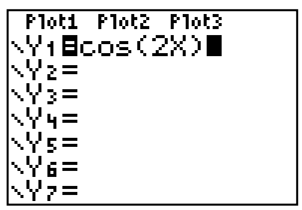
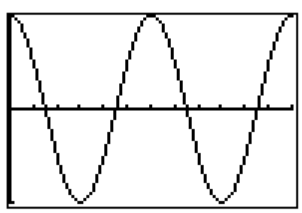
i. Draw the graphs of sine, cosine, and tangent for angles (between 0° and 360°).



EXAMPLE QUESTION :

Which of the following represents the graph of $y = \cos 2x$ *for* $0^\circ \le x \le 360^\circ$ *?*

(SPM 2005)

Step	***Procedure***	***Screenshot / key-stroke***	***Notes***
1	*Press* $\boxed{MODE}$*-key and choose DEGREE* *Press* $\boxed{ENTER}$*-key*		• *The unit we use in this example is* ***degree*** $0^\circ \le x \le 360^\circ$
2	*Press* $\boxed{WINDOW}$*-key and key in the figure:* *for* $0^\circ \le x \le 360^\circ$		• *Key in the* ***scale of 1 unit*** *of the graph of y-axis and x-axis (Xscl and Yscl)* • *Xmin, Xmax, Ymin, and Ymax are the* ***range*** *of the graph*
3	*Press* $\boxed{Y=}$*, and key in the equation:* $y = \cos 2x$ $\boxed{Y=}\boxed{COS}\boxed{2}\boxed{X,T,\Theta,n}\boxed{)}$		
4	*Press* $\boxed{GRAPH}$*-key*		• *The graph will be plotted according to the* ***range*** *fixed in step 2 (Xmin, Xmax, Ymin, and Ymax).* • *Maximum value of y-axis is* ***1*** • *Minimum value of y-axis is* ***-1*** • *Maximum value of x-axis is* ***360 degree*** • *Minimum value of x-axis is* ***0 degree***

Question for discussion.

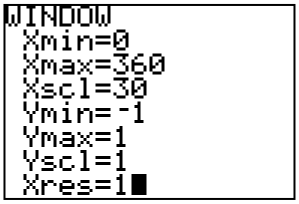
Compare the result with the graph of $y = \cos x$*. Do you see any difference?*

……

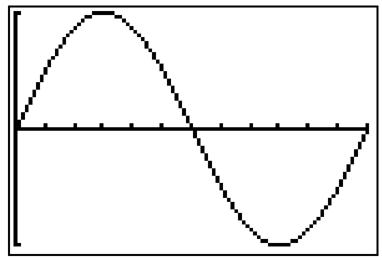
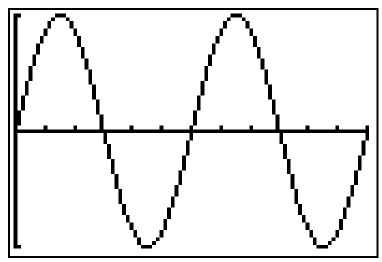
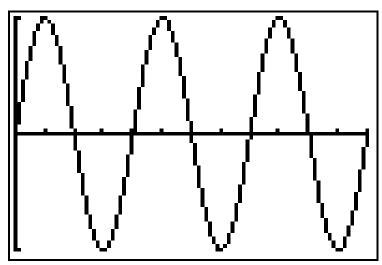
TEACHER'S NOTE

ACTIVITY 1

Use a graphing calculator to draw a graph of each function and then complete the table below



You may use this setting.

NO	TRIGONOMETRIC FUNCTION	GRAPH
1	$y = \sin x$	
2	$y = \sin 2x$	
3	$y = \sin 3x$	

Questions for discussion.

i. *What do you notice about the difference of the graphs?*

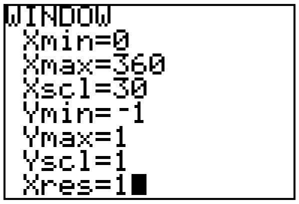
ii. *Can you predict the shape of the graph if the question asks for* $y = \sin 10x$*?*

iii. *Can you predict the shape of the graph if the question asks for* $y = \sin x/2$*?*

STUDENT'S WORKSHEET

ACTIVITY 1

Use a graphing calculator to draw a graph of each function and then complete the table below



You may use this setting.

NO	TRIGONOMETRIC FUNCTION	GRAPH
1	$y = \sin x$	
2	$y = \sin 2x$	
3	$y = \sin 3x$	

Questions for discussion.

i. *What do you notice about the difference of the graphs?*

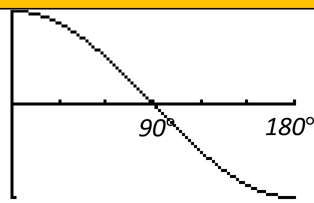
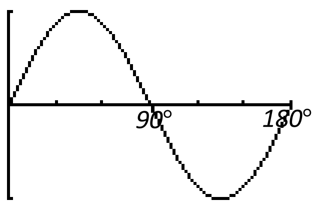
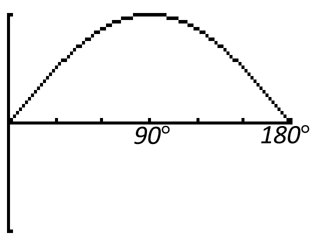
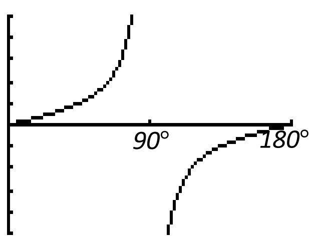
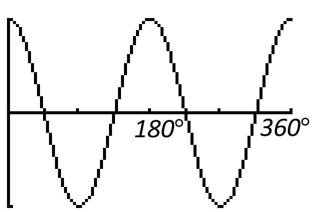
ii. *Can you predict the shape of the graph if the question asks for* $y = \sin 10x$*?*

iii. *Can you predict the shape of the graph if the question asks for* $y = \sin x/2$*?*

TEACHER'S NOTE

ACTIVITY 2

DRAW THE GRAPH THAT REPRESENTS THE TRIGONOMETRIC FUNCTION GIVEN.
(YOU MAY NEED TO HAVE THE SUITABLE WINDOW SETTING)

NO	TRIGONOMETRIC FUNCTION	GRAPH
1	$y = \cos x$ *for* $0° \leq x \leq 180°$	 *WINDOW SET:* *X-scale → 30° Y-scale → 1* *Ymin → - 1 Ymax → 1* *Xmin → 0° Xmax → 180°*
2	$y = \sin 2x$ *for* $0° \leq x \leq 180°$	 *WINDOW SET:* *X-scale → 30° Y-scale → 1* *Ymin → - 1 Ymax → 1* *Xmin → 0° Xmax → 180°*
3	$y = \sin x$ *for* $0° \leq x \leq 180°$	 *WINDOW SET:* *X-scale → 30° Y-scale → 1* *Ymin → - 1 Ymax → 1* *Xmin → 0° Xmax → 180°*
4	$y = \tan x$ *for* $0° \leq x \leq 180°$	 *WINDOW SET:* *X-scale → 90° Y-scale → 1* *Ymin → - 5 Ymax → 5* *Xmin → 0° Xmax → 180°*
5	$y = \cos 2x$ *for* $0° \leq x \leq 360°$	 *WINDOW SET:* *X-scale → 90° Y-scale → 1* *Ymin → - 1 Ymax → 1* *Xmin → 0° Xmax → 360°*

STUDENTS' WORKSHEET

ACTIVITY 2

DRAW THE GRAPH THAT REPRESENT THE TRIGONOMETRIC FUNCTION GIVEN
(YOU MAY NEED TO HAVE THE SUITABLE WINDOW SETTING)

NO	TRIGONOMETRIC FUNCTION	GRAPH
1	$y = cos\, x$ *for* $0° \leq x \leq 180°$	
2	$y = sin\, 2x$ *for* $0° \leq x \leq 180°$	
3	$y = sin\, x$ *for* $0° \leq x \leq 180°$	
4	$y = tan\, x$ *for* $0° \leq x \leq 180°$	
5	$y = cos\, 2x$ *for* $0° \leq x \leq 360°$	

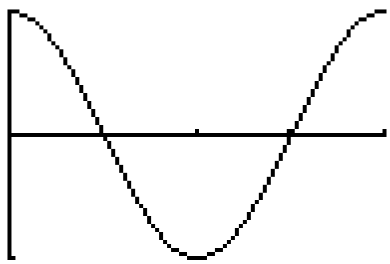
TEACHER'S NOTE

SPM EXAMPLE QUESTION

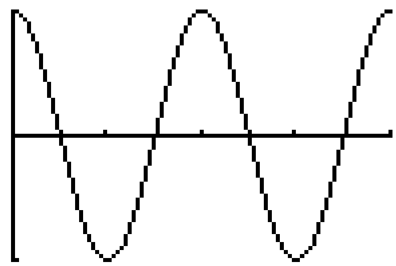
1 *Which of the following graph represent* $y = \cos 2x$ *for* $0^\circ \leq x \leq 360^\circ$ *?*

SPM 2006

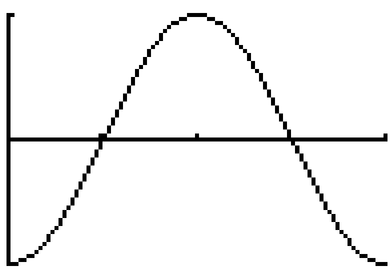
A



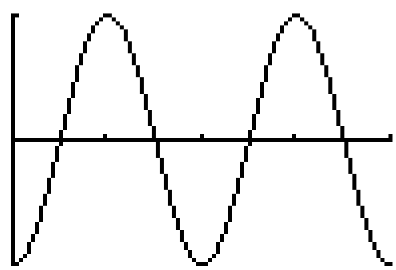
B



C



D



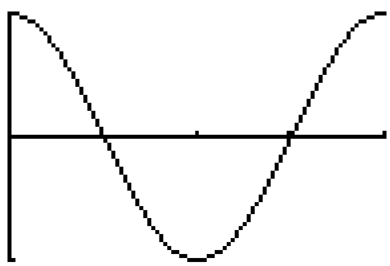
STUDENT'S WORKSHEET

SPM EXAMPLE QUESTION

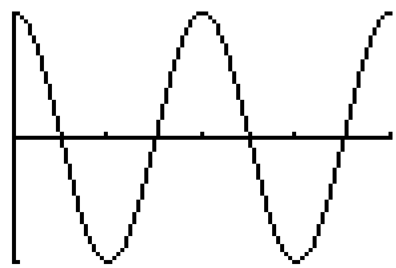
2 *Which of the following graph represent* $y = \cos 2x$ *for* $0^\circ \leq x \leq 360^\circ$*?*

SPM 2006

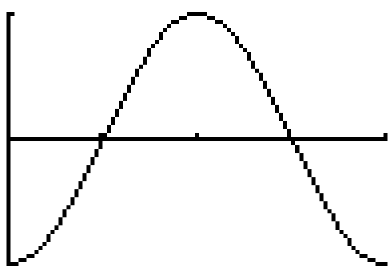
A



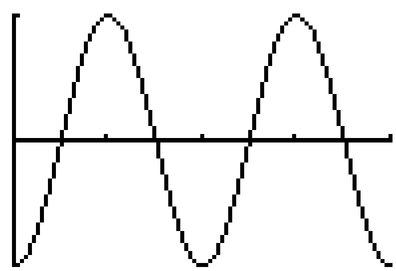
B



C



D



TOPIC **:** ***GRAPH OF FUNCTIONS 2***

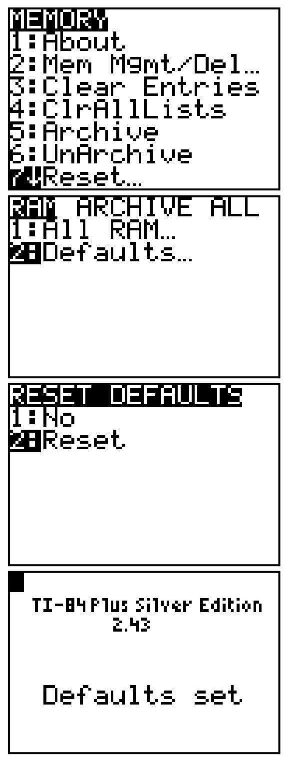
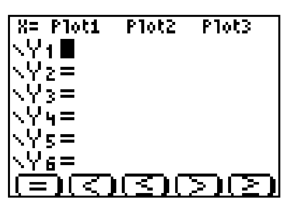
LESSON OBJECTIVE **:**

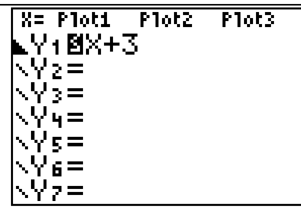
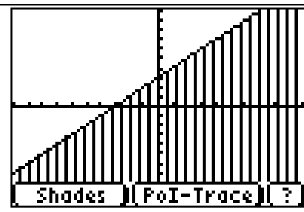
Students will be able to…

i. Shade the regions representing the inequalities

APPLICATION **:** *INEQUALZ*

EXAMPLE QUESTION **:** *Shade the region representing the inequalities,* $y \leq x + 3$

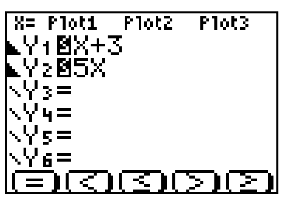
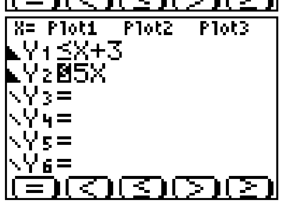
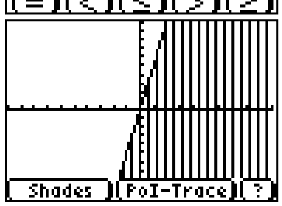
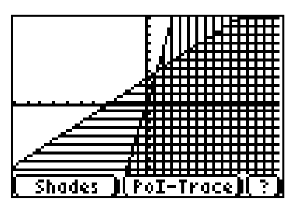
Step	***Procedure***	***Screenshot / key-stroke***	***Notes***
1	*[2nd][+] for [MEM] mode,* *Press* ***7:Reset***, ***2:Defaults***, ***2:Reset***. *Then,* ***'Defaults Set'*** *appear*		***MAKE SURE MEM IS RESET TO DEFAULT***
2	Press [APPS]; Scroll down until find **:Inequalz**. *[ENTER]*		• *The applications (apps) consisting extra programs, and need to* ***be pre-downloaded*** *before can be used.*
3	Press [Y=]		• *When the cursor is on the* ***'='*** *sign, the* ***inequalities signs*** *appear at the bottom of the screen*

<table>
<tr>
<td>4</td>
<td>Key in the linear inequalities,
$y \leq x + 3$
Press [ALPHA] [F3], to select ≤. Key in $x + 3$
[ENTER]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Press [GRAPH]</td>
<td></td>
<td>• The region representing the inequalities will be shaded</td>
</tr>
<tr>
<td colspan="4">Questions for discussion.

i. What do you notice the kind of line of the graph? Dashed or solid? What does it means by having dashed or solid line?

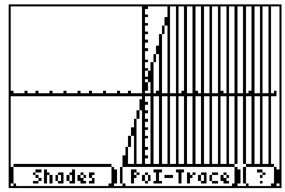
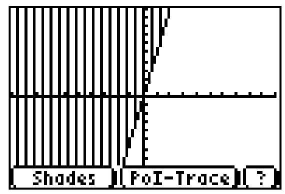
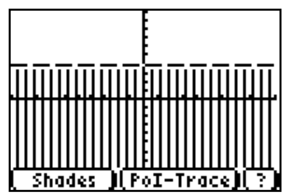
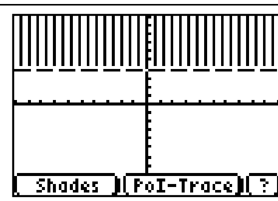
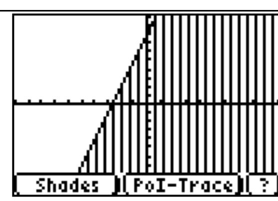
ii. What do you think will happen if the inequality is changed to $y < x + 3$?

iii. Do you see the difference between the equation $y = x + 3$, with the inequalities $y < x + 3$, $y \leq x + 3$, $y > x + 3$, or $y \geq x + 3$?

iv. Let say, to key in another inequality, $y \leq 5x$ what is the operational procedure?</td>
</tr>
<tr>
<td>6</td>
<td colspan="2"></td>
<td>• De-select the first inequalities by [ENTER] on the inequalities sign
• IF NOT, the calculator will DRAW THE GRAPH FOR ALL THE SELECTED INEQUALITIES.
</td>
</tr>
</table>

TEACHER'S NOTE

ACTIVITY 1

No	***Inequalities***	***Sketch your shaded region***
1.	$y \leq 5x$	
2.	$y \geq 5x$	
3.	$y < 4$	
4.	$y > 4$	
5.	$y \leq 3x + 8$	

Discussion:

(a) What do you notice, the difference between the shaded regions in each question 1 and 2?
The first one shaded to the right and the other one shaded to the left

(b) What do you notice, the difference between the lines in question 3 and 4?
The first one shaded to the bottom and the other one shaded up

(c) Can you make general conclusion for what you have discover?
The ($>, \geq$) signs will always shaded upper part of y-axis, and ($<, \leq$.)signs will always shaded lower part of y-axis of the function

STUDENT'S WORKSHEET

ACTIVITY 1

No	*Inequalities*	*Sketch your shaded region*
1.	$y \leq 5x$	
2.	$y \geq 5x$	
3.	$y < 4$	
4.	$y > 4$	
5.	$y \leq 3x + 8$	

Discussion:

(a) What do you notice, the difference between the shaded regions in each question 1 and 2?

(b) What do you notice, the difference between the lines in question 3 and 4?

(c) Can you make general conclusion for what you have discover?

TOPIC : ***GRAPH OF FUNCTIONS 2***

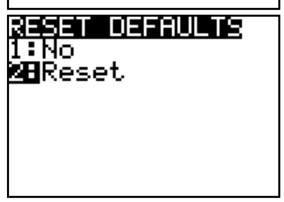
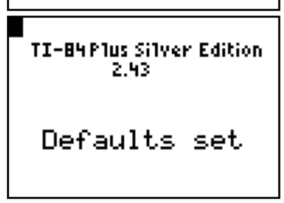
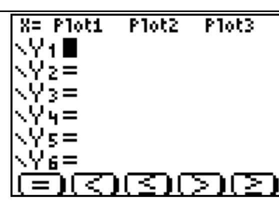
LESSON OBJECTIVE :

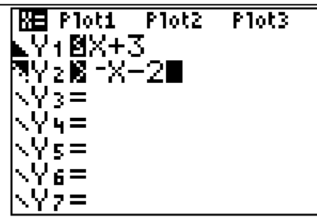
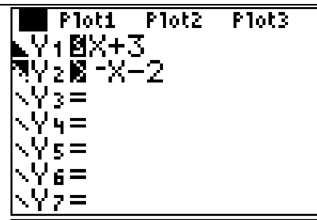
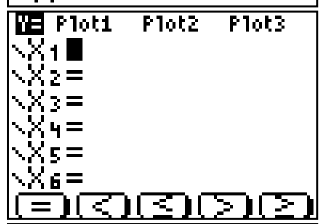
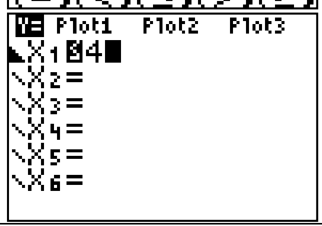
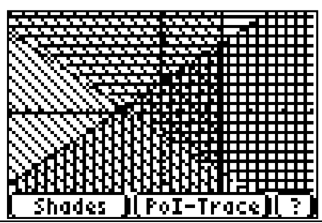
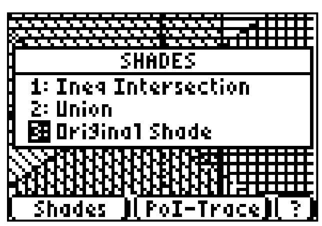
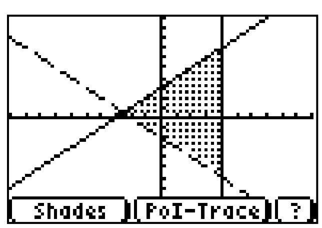
Students will be able to…

i. Determine the region which satisfies two or more simultaneous linear inequalities.

APPLICATION : *INEQUALZ*

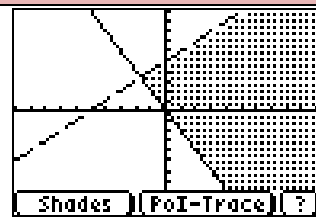
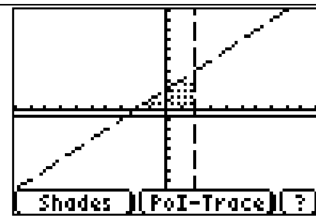
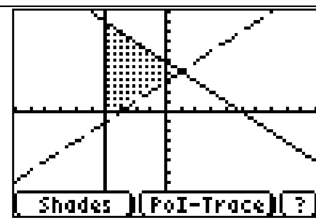
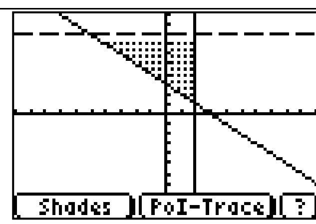
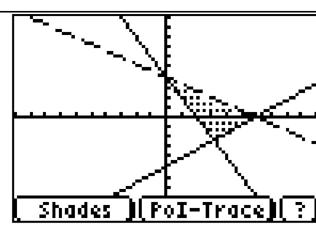
EXAMPLE QUESTION : *Shade the region representing the inequalities,* $y \leq x + 3$, $x \leq 4$, *and* $y > x - 2$

Step	*Procedure*	*Screenshot / key-stroke*	*Notes*
1	*[2nd][+] for [MEM] mode,* *Press **7:Reset**,* ***2:Defaults**,* ***2:Reset**.* *Then, **'Defaults Set'** appear*		***MAKE SURE MEM IS RESET TO DEFAULT;*** ***7:RESET*** ***MAKE SURE ALSO, ALL ENTRIES ARE GONE*** ***3:CLEAR ENTRIES*** ***RESET ALL RAM ALSO CAN BE DONE***
2	Press [APPS]-key; Scroll down until find **:Inequalz**. *[ENTER]*		• *The applications (apps) consisting extra programs, and need to **be pre-downloaded** before can be used.*
3	Press [Y=]-key		• *When the cursor is on the = sign, the **inequalities sign** appear at the bottom of the screen*

4	*Key in the linear inequalities,* $\boldsymbol{y \leq x + 3}$ **and** $\boldsymbol{y} > -x - 2$		• *Same method as before.* • *Include all **Y's inequalities***
5	*Move the cursor to the **top-left**, and press* *[ENTER]* *Key in the linear inequalities,* $\boldsymbol{x \leq 4}$		• ***The screen will be shifted to X's equations***
6	*Press* *[GRAPH]*		• *The region representing the inequalities will be shaded*
7	*Press* *[ALPHA][Y=] for [F1] mode* *Select **1:ineq intersection***		

TEACHER'S NOTE

ACTIVITY 1

No	***Inequalities***	***Sketch your shaded region***
1.	$y < x + 5$ $y \geq -2x$	
2.	$y \leq x + 2$ $y \geq \frac{1}{2}$ $x < 2$	
3.	$y \leq 5 - x$ $x \geq -4$ $x < y - 3$ *(Hint: re-arrange the inequalities)*	
4.	$x + y \geq 3$ $y < 8$ $x \leq 2$	
5.	$y \geq \frac{5}{6}x - 5$ $y < -\frac{2}{3}x + 4$ $y \geq -2x + 4$	

STUDENT'S WORKSHEET

ACTIVITY 1

No	Inequalities	Sketch your shaded region
1.	$y < x + 5$ $y \geq -2x$	
2.	$y \leq x + 2$ $y \geq \frac{1}{2}$ $x < 2$	
3.	$y \leq 5 - x$ $x \geq -4$ $x < y - 3$ *(Hint: re-arrange the inequalities)*	
4.	$x + y \geq 3$ $y < 8$ $x \leq 2$	
5.	$y \geq \frac{5}{6}x - 5$ $y < -\frac{2}{3}x + 4$ $y \geq -2x + 4$	

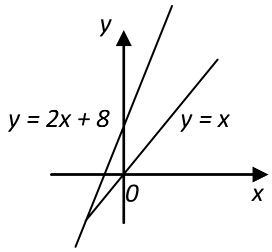
TEACHER'S NOTE

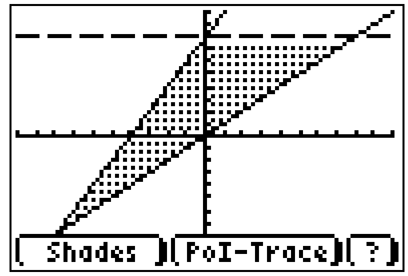
ACTIVITY 2 - *SPM QUESTIONS*

Sketch the shaded region

<table>
<tr>
<td>[SPM2003]</td>
<td>

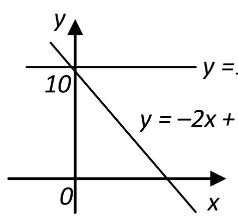
$y \leq 2x + 8$, $y \geq x$ and $y < 8$

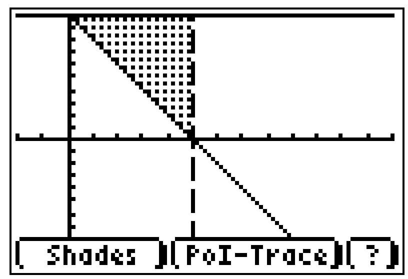


</td>
<td>


</td>
</tr>
<tr>
<td>[SPM2005]</td>
<td>

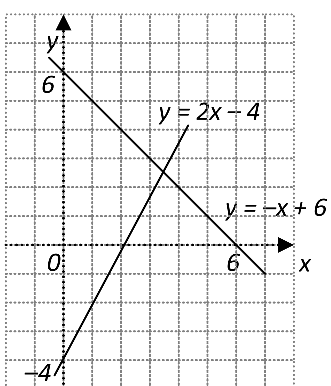
$y \geq -2x + 10$, $x < 5$ and $y \leq 10$



</td>
<td>


</td>
</tr>
<tr>
<td>[SPM2007]</td>
<td>

$y > 2x - 4$, $y < -x + 6$ and $x > 1$



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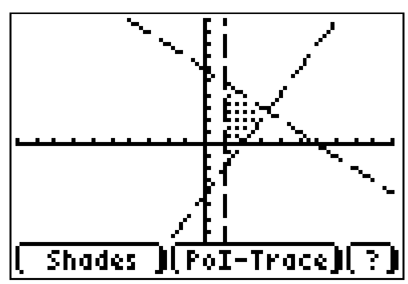
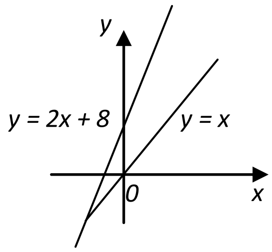
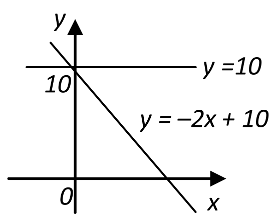
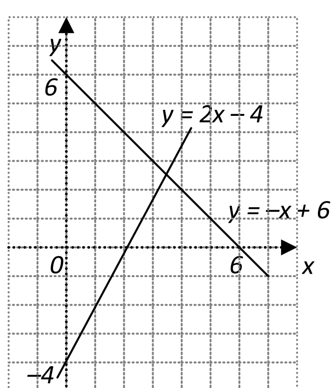


</td>
</tr>
</table>

STUDENT'S WORKSHEET

ACTIVITY 2 - *SPM QUESTIONS*
Sketch the shaded region

[SPM2003]	$y \le 2x + 8$, $y \ge x$ and $y < 8$ 	
[SPM2005]	$y \ge -2x + 10$, $x < 5$ and $y \le 10$ 	
[SPM2007]	$y > 2x - 4$, $y < -x + 6$ and $x > 1$ 	

TOPIC **:** ***MATRICES***

LESSON OBJECTIVE **:**

Students will be able to…

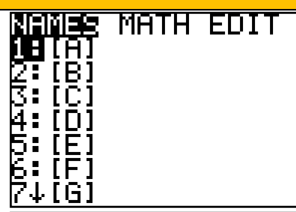
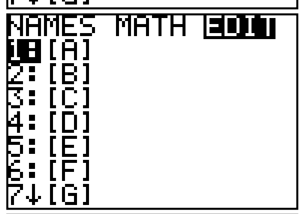
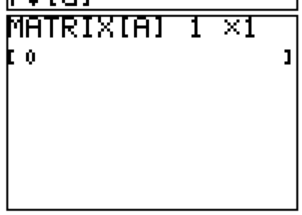
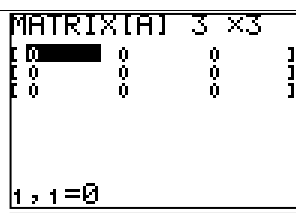
i. Understand and use the concept of matrices

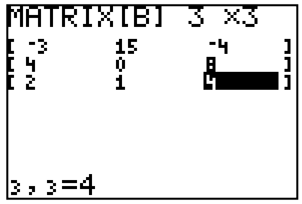
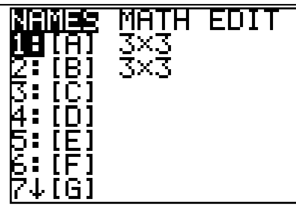
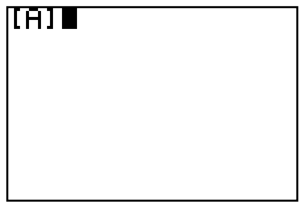
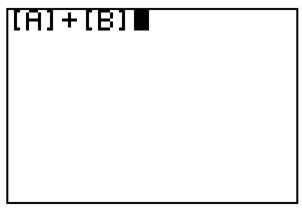
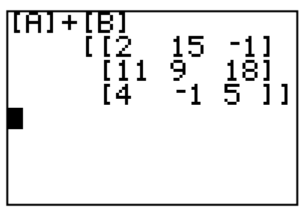
EXAMPLE QUESTION **:** *Key in the elements of matrix A and matrix B such that;*

$$A = \begin{pmatrix} 5 & 0 & 3 \\ 7 & 9 & 10 \\ 2 & -2 & 1 \end{pmatrix},\ B = \begin{pmatrix} -3 & 15 & -4 \\ 4 & 0 & 8 \\ 2 & 1 & 4 \end{pmatrix}$$

Then, find:

1) *the sum of the two matrices*
2) *the difference of the two matrices*
3) *the multiplication of the two matrices*

Step	***Procedure***	***Screenshot / key-stroke***	***Notes***
1	*Press* [2nd][x^{-1}] *for* [MATRIX] *mode* *Choose* ***EDIT 1: [A]*** [ENTER]		*Make sure to clear the memory before start.* *Step: Press* [2nd] [+] *,* *choose* ***7 : Reset*** ***1 : All RAM*** ***2 : Reset*** [ENTER]
2	*Press* [3] [ENTER] [3] [ENTER] *Press* [5] [ENTER], [0] [ENTER], [3] [ENTER] [7] [ENTER], [9] [ENTER], [1] [0] [ENTER] [2] [ENTER], [(-)] [2] [ENTER], [1] [ENTER]		• *Setting the matrix order as the question asked, which is 3 × 3* • *Then, key in the data starting with the 1st column until the end.*

3	*Repeat step 1, choose **EDIT 2: [B]** [ENTER]* *Follow the same procedure to key in the data for **Matrix B***		• *To edit the element of the matrix, simply move the cursor and redo the entry.*
4	*To find the sum of two matrices:* *Press [2nd] [MODE] for [QUIT] mode and return to **Home Screen*** *Press [2nd][x^{-1}] for [MATRIX] mode* *Choose **1:[A] 3× 3** [ENTER]* *Press [+] to perform additional operation* *Press [2nd][x^{-1}] for [MATRIX] mode* *Choose **2:[B] 3× 3** [ENTER]* *Press [ENTER] for answer*	   	• *The saved elements for each matrix is displayed* • *Use the same procedure to find **the SUBTRACTION and MULTIPLICATION.*** • *Press[CLEAR], then follow step 4 for subtraction and multiplication.* ***Question 1*** *The sum of the two matrices* ***Question 2*** *The difference of the two matrices* ***Question 3*** *the multiplication of the two matrices*

TEACHER'S NOTE

ACTIVITY 1

The table below shows the marks obtained by Ahmad in the February and March tests. Each of the test contributed 50% of the total marks for the first term.

FEBRUARY		***MARCH***	
Malay language	*32*	*Malay language*	*41*
English	*37*	*English*	*27*
Mathematics	*35*	*Mathematics*	*37*
Science	*20*	*Science*	*31*

*(a) Present the above information in **matrix form.***

*(b) Calculate the **total marks** obtained for each subject in the first term.*

Solution:

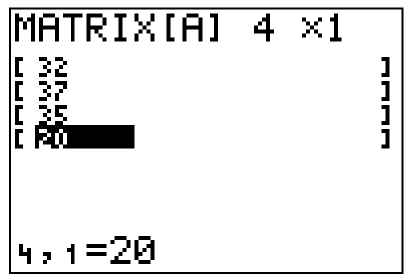
Press [2nd] [x^{-1}] for [MATRIX] mode

***EDIT 1: [A]** [ENTER]*

Press [4] [ENTER] [1] [ENTER] to set the matrix order;

Key in the mark for February:

[3] [2] [ENTER], [2] [7] [ENTER], [3] [5] [ENTER], [2] [0] [ENTER]



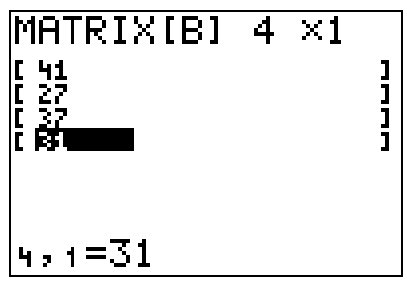
Press [2nd] [x^{-1}] for [MATRIX] mode

***EDIT2: [B]** [ENTER]*

Press [4] [ENTER] [1] [ENTER] to set the matrix order;

Key in the mark for March:

[4] [1] [ENTER], [2] [9] [ENTER], [3] [7] [ENTER],[3] [1] [ENTER]



*Press [2nd] [MODE] for [QUIT] mode and return to **Home Screen***

Press [2nd] [x^{-1}] for [MATRIX] mode

*Choose **NAME***

***1: [A]** [ENTER]*

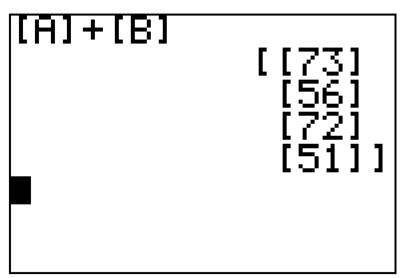
Press [+] key

Press [2nd] [x^{-1}] for [MATRIX] mode

*Choose **NAME***

***1: [B]** [ENTER]*

[ENTER]



STUDENT'S WORKSHEET

ACTIVITY 1

The table below shows the marks obtained by Ahmad in the February and March tests. Each of the test contributed 50% of the total marks for the first term.

FEBRUARY		***MARCH***	
Malay language	*32*	*Malay language*	*41*
English	*37*	*English*	*27*
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Science	*20*	*Science*	*31*

(a) *Present the above information in **matrix form.***

(b) *Calculate the **total marks** obtained for each subject in the first term.*

Solution:

TEACHER'S NOTE

ACTIVITY 2

Given matrix A and B as below:

$$A = \begin{pmatrix} 2 & 3 \\ 5 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 10 & 3 \\ 1 & 7 \end{pmatrix}$$

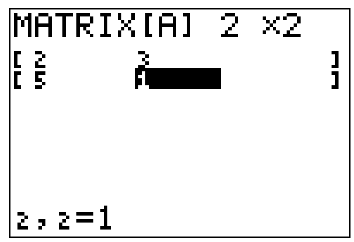
Find

a) AB
b) BA

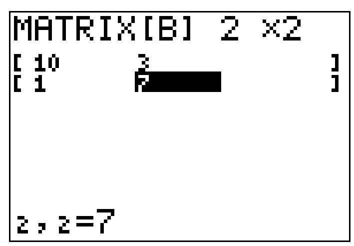
DISCUSSION:
- *What can you say about the relation between **AB** and **BA**?*

Solution:

Key in the data for matrix A.

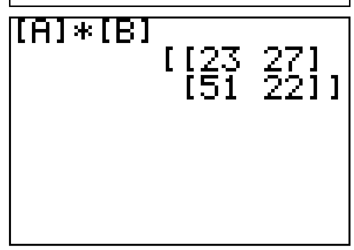


Repeat the step above to enter the value in matrix B

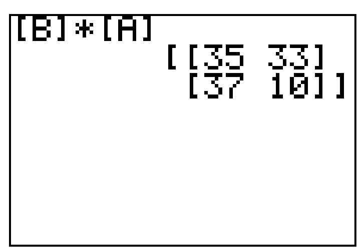


*Press [2nd] [MODE] for [QUIT] mode and return to **Home Screen***

Press [2nd][x^{-1}] for [MATRIX] mode
*Choose **NAME***
***1: [A]** [ENTER]*
Press [×] key
Press [2nd][x^{-1}] for [MATRIX] mode
*Choose **NAME***
***1: [B]** [ENTER]*
[ENTER]



Repeat the same procedure for [B] × [A]



STUDENT'S WORKSHEET

ACTIVITY 2

Given matrix A and B as below:

$$A = \begin{pmatrix} 2 & 3 \\ 5 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 10 & 3 \\ 1 & 7 \end{pmatrix}$$

Find

a) AB
b) BA

DISCUSSION:

- *What can you say about the relation between **AB** and **BA**?*

Solution:

TEACHER'S NOTE

ACTIVITY 3

The table below shows the mass of sugar, salt and flour in kg, which is bought by 3 restaurant owners on a certain day.

	Sugar	Salt	Flour
Wan	4	1	7
Erni	3	2	6
Fauliza	5	1	9

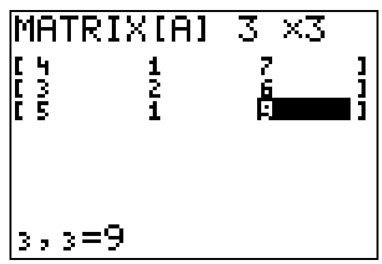
The price of sugar, salt and flour (per kg) on Saturday are as shown below:

	Price per Kg (RM)
Sugar	2.10
Salt	0.80
Flour	2.50

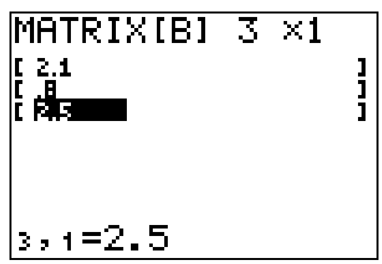
a) Key in the information given in matrix form.
b) How much does each restaurant owner spend on Saturday by using matrix?

Solution:

Key in the data for matrix A.

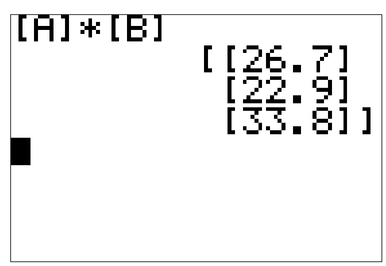


Repeat the step above to enter the value in matrix B



*Press [2nd] [MODE] for [QUIT] mode and return to **Home Screen***

Press [2nd][x^{-1}] for [MATRIX] mode
*Choose **NAMES***
***1: [A]** [ENTER]*
Press [×] key
Press [2nd][x^{-1}] for [MATRIX] mode
*Choose **NAMES***
***1: [B]** [ENTER]*
[ENTER]



STUDENT'S WORKSHEET

ACTIVITY 3

The table below shows the mass of sugar, salt and flour in kg, which is bought by 3 restaurant owners on a certain day.

	Sugar	Salt	Flour
Wan	4	1	7
Erni	3	2	6
Fauliza	5	1	9

The price of sugar, salt and flour (per kg) on Saturday are as shown below:

	Price per Kg (RM)
Sugar	2.10
Salt	0.80
Flour	2.50

1) *Key in the information given in matrix form.*
2) *How much does each restaurant owner spend on Saturday by using matrix?*

Solution:

TEACHER'S NOTE

ACTIVITY 4

*Determine whether matrix **B** is an inverse matrix of **A**.*

a) $A = \begin{pmatrix} 4 & -2 \\ 2 & 3 \end{pmatrix}$ $B = \begin{pmatrix} 3 & 2 \\ -2 & 4 \end{pmatrix}$

b) $A = \begin{pmatrix} 4 & 7 \\ 1 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 2 & -7 \\ -1 & 4 \end{pmatrix}$

DISCUSSION:

i. What is the condition for the existence of inverse matrix?

Solution:

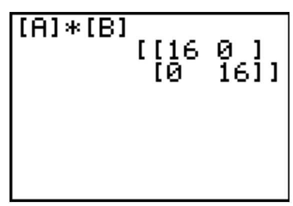
a)

Key in the data for matrix A.

Repeat the step above to enter the value in matrix B

*Press [2nd] [MODE] for [QUIT] mode and return to **Home Screen***

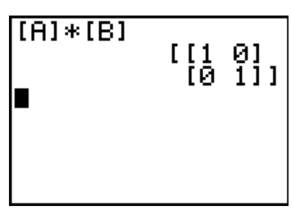
Press [2nd][x^{-1}] for [MATRIX] mode
*Choose **NAMES***
***1: [A]** [ENTER]*
Press [×] key
Press [2nd][x^{-1}] for [MATRIX] mode
*Choose **NAMES***
***1: [B]** [ENTER]*
[ENTER]



*CONCLUSION: MATRIX B IS **NOT AN INVERSE MATRIX** OF A*

b)

*CONCLUSION: MATRIX B IS **AN INVERSE MATRIX** OF A*



STUDENT'S WORKSHEET

ACTIVITY 4

*Determine whether matrix **B** is an inverse matrix of **A**.*

a) $A = \begin{pmatrix} 4 & -2 \\ 2 & 3 \end{pmatrix}$ $B = \begin{pmatrix} 3 & 2 \\ -2 & 4 \end{pmatrix}$

b) $A = \begin{pmatrix} 4 & 7 \\ 1 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 2 & -7 \\ -1 & 4 \end{pmatrix}$

DISCUSSION:

i. ***What is the condition for the existence of inverse matrix?***

Solution:

TEACHER'S NOTE

ACTIVITY 5

Determine which of the following matrix is an inverse matrix of $A = \begin{pmatrix} 4 & 2 \\ -3 & -2 \end{pmatrix}$.

$B = \begin{pmatrix} -3 & 4 \\ 1 & -2 \end{pmatrix}$, $C = \begin{pmatrix} -2 & 4 \\ 1 & -3 \end{pmatrix}$, $D = \begin{pmatrix} 1 & 1 \\ \frac{-3}{2} & -2 \end{pmatrix}$, $E = \begin{pmatrix} \frac{1}{2} & -3 \\ 1 & 7 \end{pmatrix}$

DISCUSSION:

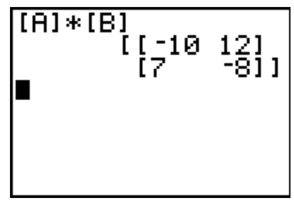
i. How to identify which matrix is the inverse of the matrix A?

Solution:

Key in the data for matrix A.

Repeat the step above to enter the value in matrix B

Press [2nd] [MODE] for [QUIT] mode and return to ***Home Screen***



Press [2nd][x^{-1}] for [MATRIX] mode

Choose ***NAME***

1: [A] *[ENTER]*

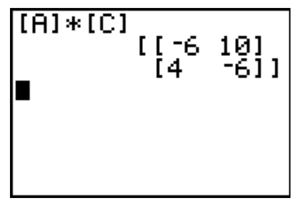
Press [×] key

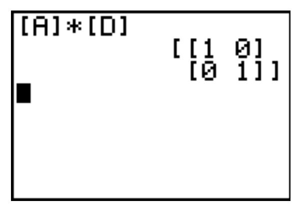
Press [2nd][x^{-1}] for [MATRIX] mode

Choose ***NAME***

1: [B] *[ENTER]*

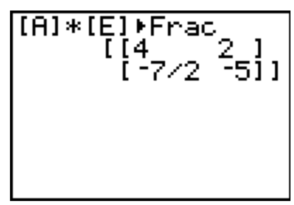
[ENTER]





CONCLUSION:

- ***MATRIX B, C and E IS <u>NOT AN INVERSE MATRIX</u> OF A***
- ***MATRIX D IS <u>AN INVERSE MATRIX</u> OF A***



STUDENT'S WORKSHEET

ACTIVITY 5

Determine which of the following matrix is an inverse matrix of $A = \begin{pmatrix} 4 & 2 \\ -3 & -2 \end{pmatrix}$.

$B = \begin{pmatrix} -3 & 4 \\ 1 & -2 \end{pmatrix}$, $C = \begin{pmatrix} -2 & 4 \\ 1 & -3 \end{pmatrix}$, $D = \begin{pmatrix} 1 & 1 \\ \frac{-3}{2} & -2 \end{pmatrix}$, $E = \begin{pmatrix} \frac{1}{2} & -3 \\ 1 & 7 \end{pmatrix}$

DISCUSSION:

i. How to identify which matrix is the inverse of the matrix A?

Solution:

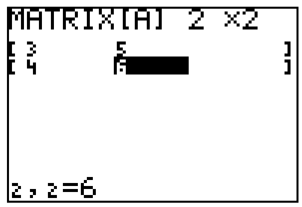
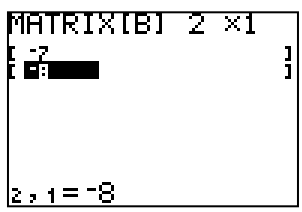
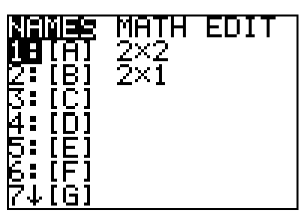
TOPIC **:** ***MATRICES***

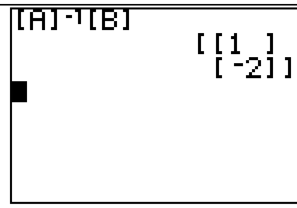
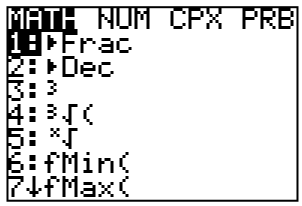
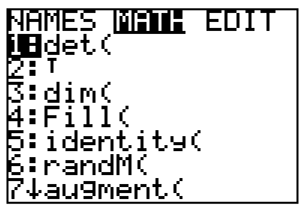
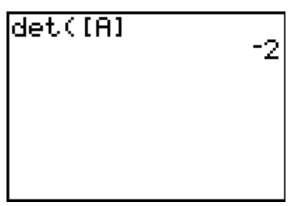
LESSON OBJECTIVE **:**

Students will be able to…

i. Understand and use the concept of inverse matrix.

EXAMPLE QUESTION **:** *Solve* $\begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -7 \\ -8 \end{pmatrix}$

Step	*Procedure*	*Screenshot / key-stroke*	*Notes*
1	*Let* ***A*** *=* $\begin{pmatrix} \mathbf{3} & \mathbf{5} \\ \mathbf{4} & \mathbf{6} \end{pmatrix}$		*So, the equation above can be write as* $A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -7 \\ -8 \end{pmatrix}$
Questions for Discussion: *i. Why the use of inverse matrix is necessary?* *ii. What happen when matrix* $A^{-1} \times$ *matrix A?*			
2	*Let* ***B*** *=* $\begin{pmatrix} -7 \\ -8 \end{pmatrix}$		*To Find* $\begin{pmatrix} x \\ y \end{pmatrix}$*, multiply inverse matrix,* A^{-1} *in both sides.* $A^{-1} A\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1}\begin{pmatrix} -7 \\ -8 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1}\begin{pmatrix} -7 \\ -8 \end{pmatrix}$ *So, key in* $\begin{pmatrix} -7 \\ -8 \end{pmatrix}$ *as matrix B*
3	*Press* $\boxed{\text{2nd}}$ $\boxed{\text{MODE}}$ *for [QUIT] mode.* *Return to* ***Home Screen*** *Press* $\boxed{\text{2nd}}\boxed{x^{-1}}$ *for* $\boxed{\text{MATRX}}$ *mode.*		

	Choose **NAMES 1: [A]** [x^{-1}] Press [2nd] [MATRX] for [MATRX] mode again and choose **NAMES** **2: [B]** [ENTER]		What is the value of x and y? $\boldsymbol{x} =$ and $\boldsymbol{y} =$
4	Some answers can be in **decimals** form; To display it in **FRACTION** form, Press [MATH]-key Choose **1:Frac** [ENTER]		
5	To calculate the **DETERMINANT ,** Press [2nd] [MATRX] for [MATRX], choose **MATH,** choose **1: det(** [ENTER] Press [2nd] [MATRX] for [MATRX], choose **NAMES,** **choose 1: [A]** [ENTER] Press [ENTER] for answer.	 	If matrix A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then, Determinant of matrix A =

Questions for discussion.

1) What is the use of determinant?

2) Can you represent simultaneous linear equations as matrix equation?

3) What rules do you follow in order to represent simultaneous linear equations as matrix equation?

TEACHER'S NOTE

ACTIVITY 1

Solve the problem given.

(a) *It is given that* $\begin{pmatrix} 1 & 2 \\ \frac{1}{2} & n \end{pmatrix}$ *is the inverse matrix of* $\begin{pmatrix} 3 & -4 \\ -1 & 2 \end{pmatrix}$. *Find the value of n.*

(b) *Write the following simultaneous linear equations as matrix equation:*

$$3u - 4v = -5$$
$$-u + 2v = 2$$

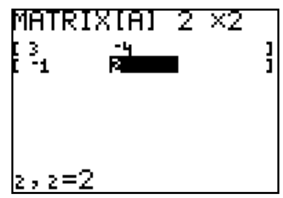
Hence, using matrices, calculate the value of u and v.

(SPM 2006)

Solution:

(a)

Key in the data for matrix A.



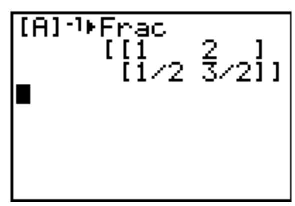
Press [2nd] [MODE] for [QUIT] mode and return to ***Home Screen***

Press [2nd] [MATRX] for [MATRX] mode again and choose ***NAMES***

1: [A]

[ENTER]

[x^{-1}]



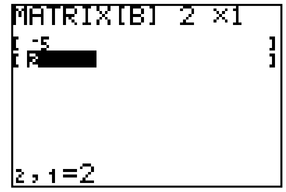
Press [MATH], choose 1: Frac [ENTER]

[ENTER]

Compare the answer with the given inverse matrix, $\therefore n = \frac{3}{2}$

(b)

Repeat the step above to enter the elements in matrix B



*Press [2nd] [MODE] for [QUIT] mode and return to **Home Screen***

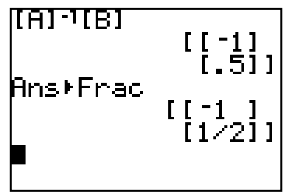
*Press [2nd][x^{-1}] for [MATRX] mode and choose **NAMES 1: [A]***

[x^{-1}]

*Press [2nd] [x^{-1}] for [MATRX] mode again and choose **NAMES***

2: [B]

[ENTER]



Press [MATH]

*Choose **1:Frac***

[ENTER]

STUDENT'S WORKSHEET

ACTIVITY 1

Solve the problem given.

(a) *It is given that* $\begin{pmatrix} 1 & 2 \\ \frac{1}{2} & n \end{pmatrix}$ *is the inverse matrix of* $\begin{pmatrix} 3 & -4 \\ -1 & 2 \end{pmatrix}$. *Find the value of n.*

(b) *Write the following simultaneous linear equations as matrix equation:*

$$3u - 4v = -5$$
$$-u + 2v = 2$$

Hence, using matrices, calculate the value of u and v.

(SPM 2006)

Solution:

STUDENT'S WORKSHEET

ENRICHMENTS

SPM 2005

It is given that matrix P = $\begin{pmatrix} 2 & -5 \\ 1 & 3 \end{pmatrix}$ *and matrix Q = k* $\begin{pmatrix} 3 & h \\ -1 & 2 \end{pmatrix}$ *such that PQ =* $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

a) *Find the value of k and of h.*
b) *Using matrices, find the value of x and of y that satisfy the following simultaneous linear equations:*

$$2x - 5y = -17$$
$$x + 3y = 8$$

SPM 2007

a) *Given* $\frac{1}{m}\begin{pmatrix} -4 & 2 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} n & -2 \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$*, find the value of m and of n.*
b) *Using matrices, calculate the value of x and of y that satisfy the following matrix equation:*

$$\begin{pmatrix} -4 & 2 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

SPM 2008

The inverse matrix of $\begin{pmatrix} 2 & 3 \\ 4 & 7 \end{pmatrix}$ *is* $\frac{1}{k}\begin{pmatrix} 7 & -3 \\ m & 2 \end{pmatrix}$.

a) *Find the value of m and of* k.
b) *Write the following simultaneous linear equations as matrix equation:*

$$2x + 3y = -1$$
$$4x + 7y = 5$$

Hence, using matrix method, calculate the value of x and of y.

TOPIC : GRADIENT AND AREA UNDER THE GRAPH

LESSON OBJECTIVE :

Students will be able to…

i. Understand and use the concept of quantity represented by the gradient of the graph.

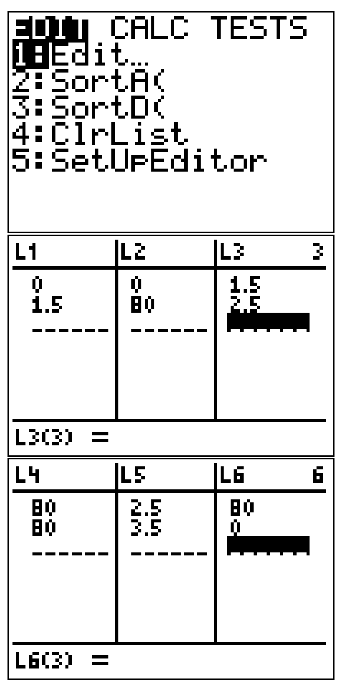
ii. Find the speed for a period of time from a distance-time graph.

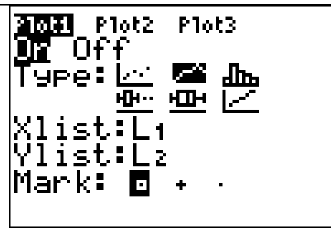
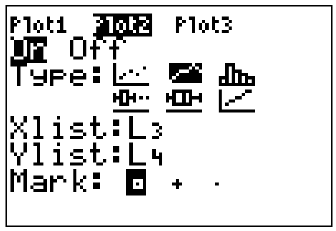
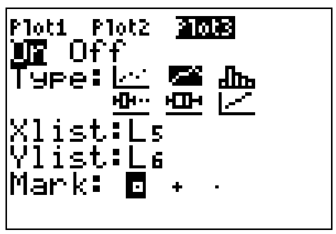
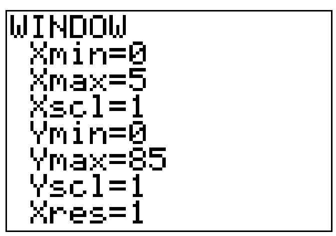
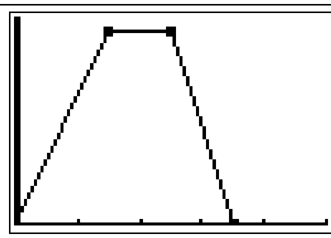
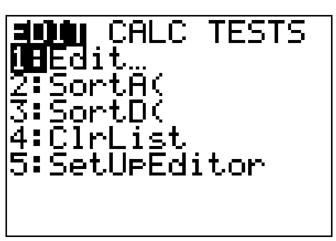
EXAMPLE QUESTION :

Ms. Devi leaves her house at 1000 to visit her friend by car. She reached home at 1330 after met her friend. Ms. Devi's journey description as below:
*Ms. Devi had drive **80 km** for **1 hour and 30 minutes** to reach her friend's house.*
*Ms. Devi took only **1 hour** to drive **back to her house**.*
Then,

a) Plot a Distance-Time graph to shows Ms. Devi's journey.
b) Calculate the speed of Ms. Devi's car for the first 1 hour.
c) Find the speed of Ms. Devi's car from 1130 to 1230.
d) Find the speed of Ms. Devi's car when going back home.

<table>
<tr><th>Step</th><th>Procedure</th><th>Screenshot / key-stroke</th><th>Notes</th></tr>
<tr><td colspan="4"><u>Understand the word problem</u> and translate it into <u>distance-time table</u> for every movement or particular time-frame.</td></tr>
<tr><td>1</td><td>Press [STAT]
Choose 1: Edit…

Key in the data for L1 and L2, L3 and L4, L5 and L6</td><td>
</td><td>Key in data as follows:
• Ms. Devi had drive <u>80 km</u> for <u>1 hour and 30 minutes</u> to reach her friend's house.
<table><tr><th>L1 (hour)</th><th>L2 (km)</th></tr><tr><td>0</td><td>0</td></tr><tr><td>1.5</td><td>80</td></tr><tr><th>L3 (hour)</th><th>L4 (km)</th></tr><tr><td>1.5</td><td>80</td></tr><tr><td>2.5</td><td>80</td></tr></table>
• Ms. Devi took only <u>1 hour</u> to drive <u>back to her house</u>
<table><tr><th>L5 (hour)</th><th>L6 (km)</th></tr><tr><td>2.5</td><td>80</td></tr><tr><td>3.5</td><td>0</td></tr></table></td></tr>
</table>

2	*Press* [2nd] [Y=] *for* [STAT PLOT], *Choose* ***1: Plot 1*** [ENTER] *For On press* [ENTER] *Select* ***Type:***, [ENTER] *Xlist:* ***L1***, *Ylist:* ***L2***		
	Press [2nd] [Y=] *for* [STAT PLOT], *Choose* ***2: Plot 2*** [ENTER] *For On press* [ENTER] *Select* ***Type:***, [ENTER] *Xlist:* ***L3***, *Ylist:* ***L4***		*To change the* ***Xlist*** *,* ***Press*** [2nd][3] *for* [L3] → [ENTER] *To change the* ***Ylist***, ***Press***[2nd][4] *for* [L4] → [ENTER]
	Press [2nd] [Y=] *for* [STAT PLOT], *Choose* ***3: Plot 3*** [ENTER] *For On press* [ENTER] *Select* ***Type:***, [ENTER] *Xlist:* ***L5***, *Ylist:* ***L6***		*To change the* ***Xlist*** *,* ***Press*** [2nd][5] *for* [L5] → [ENTER] *To change the* ***Ylist***, ***Press*** [2nd][6] *for* [L6] → [ENTER]
3	*Press* [WINDOW] *and key in the setting.*		
4	*Press* [GRAPH]		*Distance-Time graph plotted.*
5	*Press* [STAT]		

	*Select **CALC*** *Choose**4:LinReg(ax+b)*** *[ENTER]* *Press [2nd] [1] for [L1] [,] [2nd] [2] for [L2]* *[ENTER]*	EDIT CALC TESTS 1:1-Var Stats 2:2-Var Stats 3:Med-Med 4:LinReg(ax+b) 5:QuadReg 6:CubicReg 7↓QuartReg LinReg(ax+b) L_1, L_2 LinReg y=ax+b a=53.33333333 b=0	***LinReg (ax+b)** mode is an order to simulate the straight line equation, $y = mx + c$, where $m = a$, and $c = b$.* *From **LinReg**, the gradient,* *m is* *From the gradient**, the speed of Ms. Devi's car for the first 1 hour is*** $\underline{\mathbf{km/h}}$
6	*Press [STAT]* *Select **CALC*** *Choose**4:LinReg(ax+b)**[ENTER]* *Press [2nd] [3] for [L3] [,]* *[2nd] [4] for [L4]* *[ENTER]* *Press [STAT]* *Select **CALC*** *Choose**4:LinReg(ax+b)**[ENTER]* *Press [2nd] [5] for [L5] [,]* *[2nd] [6] for [L6]* *[ENTER]*	LinReg y=ax+b a=0 b=80 LinReg y=ax+b a=-80 b=280	*From LinReg, the gradient, m is 0.* *We can conclude that, the gradient, which is **the speed of Ms. Devi's car from 1130 to 1230 is*** $\underline{\mathbf{km/h}}$ *From LinReg, the gradient, m is* *We can conclude that, the gradient, which is **the speed of Ms. Devi's car when going back home is*** $\underline{\mathbf{km/h}}$

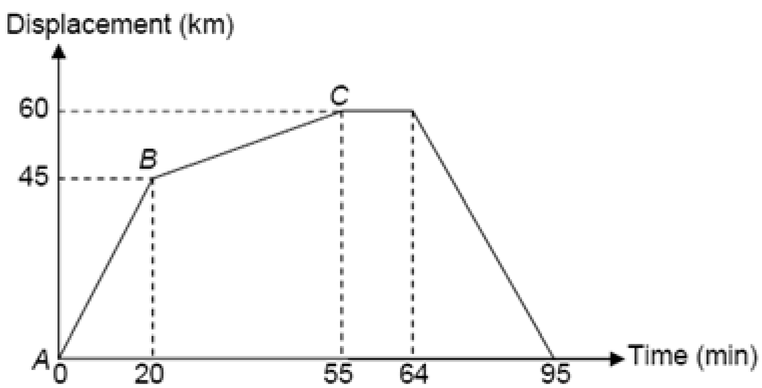
Discussion:

1. *From the graph, what can you say about Ms. Devi's car from 1130 to 1230?*

2. *Can you give the meaning for negative sign of a gradient?*

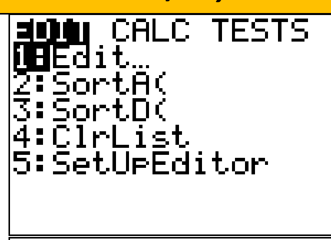
ACTIVITY 1

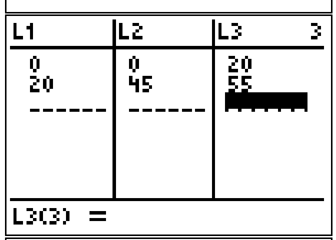
Diagram shows a displacement-time graph for the journey of a car from town A to town C passing town B then back to town A.

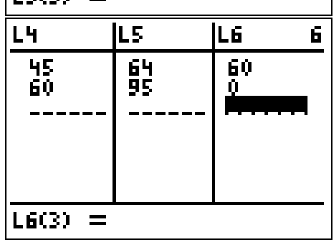


(a) Calculate the speed in km/h for the journey from town A to town B.
(b) Calculate the speed in km/h for the journey from town B to town C.
(c) Calculate the speed of the car when going back to town A.

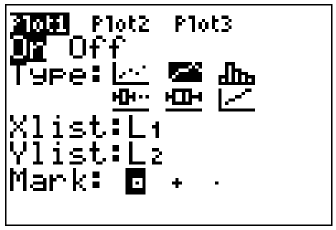
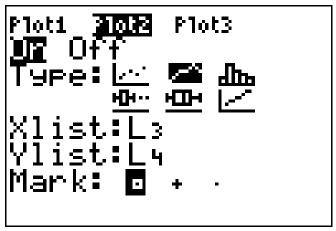
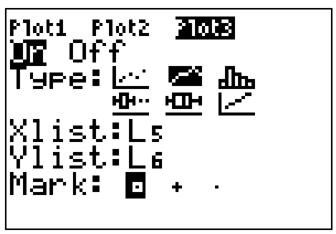
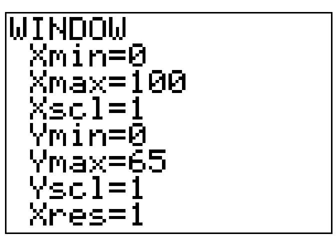
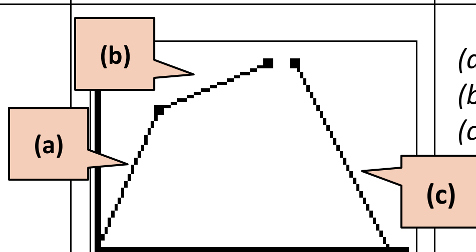
ANSWER:

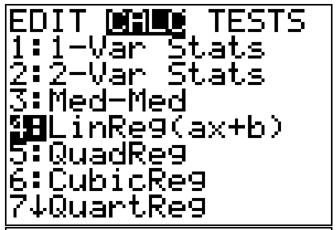
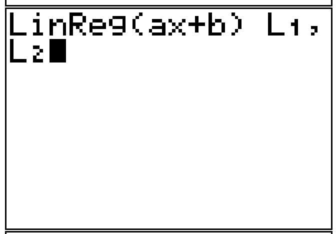
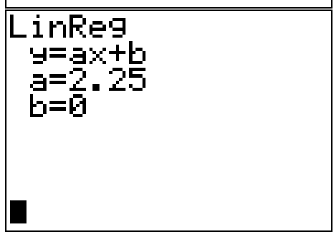
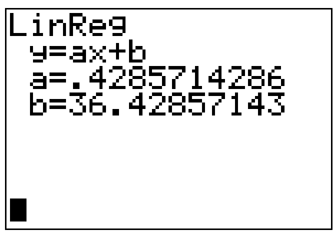
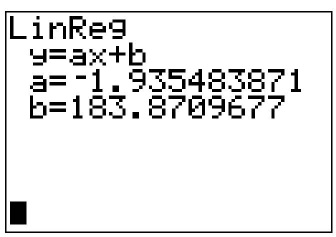
<table>
<tr><th>Step</th><th>Procedure</th><th>Screenshot / key-stroke</th><th>Notes</th></tr>
<tr>
<td>1</td>
<td>Press [STAT]
Choose 1:Edit…</td>
<td>




</td>
<td>Construct the graph from town A to town B and from town B to town C to find the speed.

(a)
<table>
<tr><td>L1</td><td>Time (min)</td><td>0</td><td></td></tr>
<tr><td>L2</td><td>Distance (km)</td><td>0</td><td></td></tr>
</table>
(b)
<table>
<tr><td>L3</td><td>Time (min)</td><td>20</td><td>55</td></tr>
<tr><td>L4</td><td>Distance (km)</td><td></td><td></td></tr>
</table>
(c)
<table>
<tr><td>L5</td><td>Time (min)</td><td></td><td></td></tr>
<tr><td>L6</td><td>Distance (km)</td><td>60</td><td>0</td></tr>
</table></td>
</tr>
</table>

2	(a) Town A to town B Press [2nd] [Y=] for [STAT PLOT], Choose **1: Plot 1** [ENTER] → **On** → [ENTER] Select **Type:** , [ENTER] Xlist: **L1**, Ylist: **L2**		
	(b) Town B to town C Press [2nd] [Y=] for [STAT PLOT], Choose **2: Plot 2** [ENTER] For On press [ENTER] Select **Type:** , [ENTER] Xlist: **L3**, Ylist: **L4**		To change the **Xlist** , **Press** [2nd] [3] for [L3] → [ENTER] To change the **Ylist**, **Press** [2nd] [4] for [L4] → [ENTER]
	(c) Going back to town A Press [2nd] [Y=] for [STAT PLOT], Choose **3: Plot 3** [ENTER] For On press [ENTER] Select **Type:** , [ENTER] Xlist: **L5**, Ylist: **L6**		To change the **Xlist** , **Press** [2nd] [5] for [L5] → [ENTER] To change the **Ylist**, **Press** [2nd] [6] for [L6] → [ENTER]
3	Press [WINDOW] and key in the setting.		
4	Press [GRAPH]		(a) Graph from town A to town B. (b) Graph from town B to town C. (c) Going back to town A

5	*Press* [STAT] *Choose* ***CALC*** *Select* ***4:LinReg(ax+b)*** *Press* [2nd][1] *for* [L1] [,] [2nd][2] *for* [L2] [ENTER]	  	*(a) The speed from town A to town B is* $km/min.$ $\times$ 60 min = $\mathbf{km/h}$ ***The speed from town A to town B is*** $\mathbf{km/h.}$
6	*Press* [STAT] *Select* ***CALC*** *Choose* ***4:LinReg(ax+b)*** [ENTER] *Press* [2nd][3] *for* [L3] [,] [2nd][4] *for* [L4] [ENTER]		*(b)* *The speed from town B to town C is* $km/min.$ $\times$ 60 min = $\mathbf{km/h}$ ***The speed from town B to town C is*** $\mathbf{km/h.}$
7	*Press* [STAT] *Select* ***CALC*** *Choose* ***4:LinReg(ax+b)*** [ENTER] *Press* [2nd][5] *for* [L5] [,] [2nd][6] *for* [L6] [ENTER]		*(c)* *The speed going back to town A is* $km/min.$ $\times$ 60 min = $\mathbf{km/h}$ ***The speed going back to town A is*** $\mathbf{km/h.}$

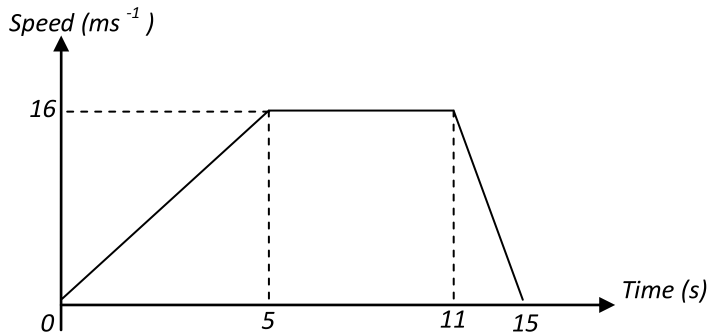
Questions for discussion

i. What is the gradient of a graph represents?

ii. Can you tell the difference between distance-time graph and speed-time graph?

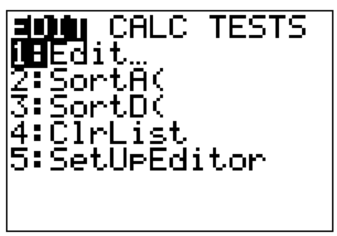
ACTIVITY 2

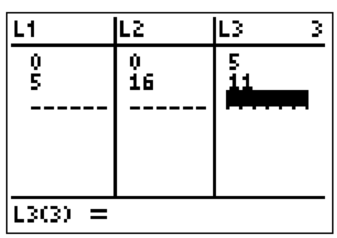
Diagram shows speed-time graph of a particle for a period of 15 seconds.

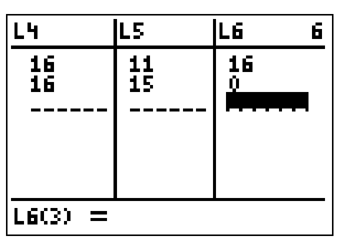


a) *Calculate the **distance, in m**, for the first 5 seconds.*
b) *Calculate the **rate of change of speed, in m s** $^{-2}$, in the **first 5 seconds***
c) *Calculate the **rate of change of speed, in ms** $^{-2}$, in the **last 4 seconds**.*

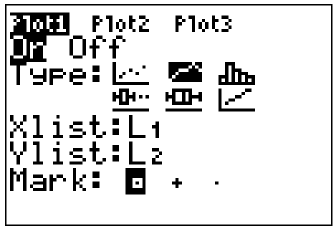
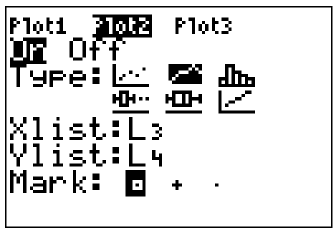
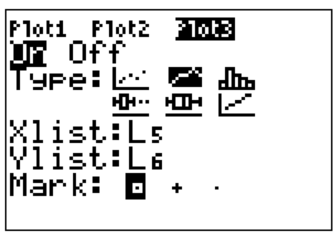
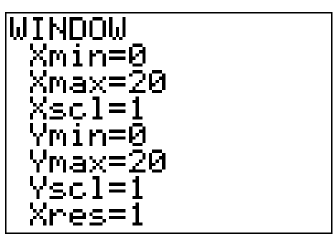
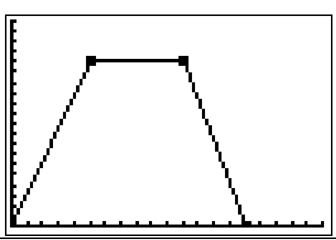
ANSWER:

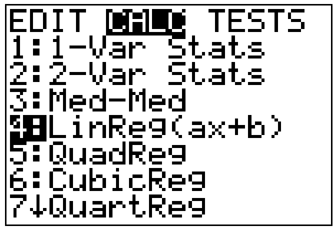
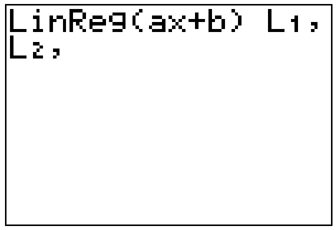
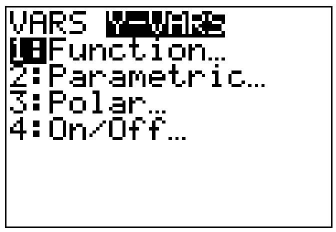
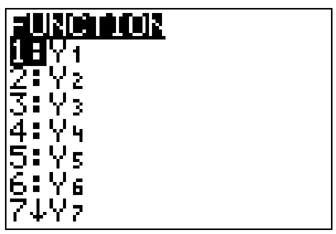
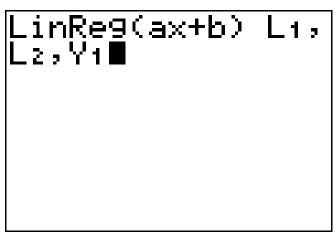
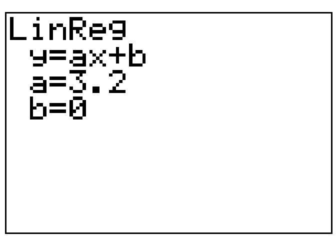
<table>
<tr><th>Step</th><th>Procedure</th><th>Screenshot / key-stroke</th><th>Notes</th></tr>
<tr><td>1</td><td>Press STAT
Choose 1:Edit…</td><td>

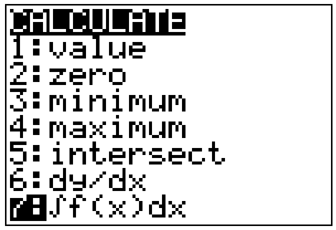
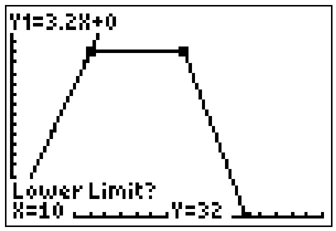
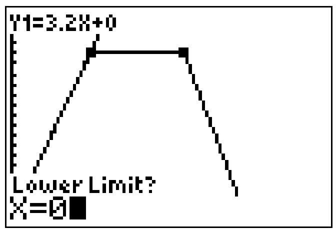
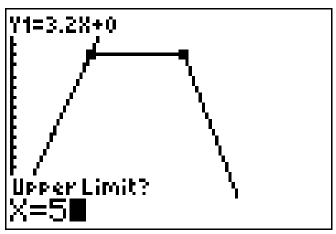
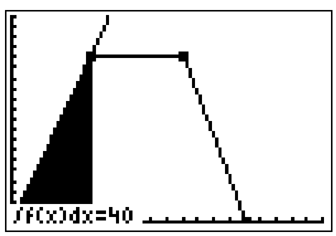
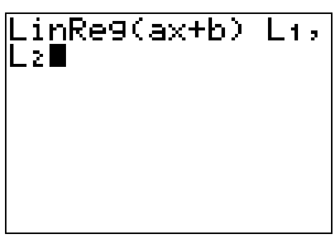



</td><td>Construct the graph from town A to town B and from town B to town C to find the speed.

(a)<table><tr><td>L1</td><td>Time (min)</td><td></td><td></td></tr><tr><td>L2</td><td>Distance (km)</td><td></td><td></td></tr></table>(b)<table><tr><td>L3</td><td>Time (min)</td><td></td><td></td></tr><tr><td>L4</td><td>Distance (km)</td><td></td><td></td></tr></table>(c)<table><tr><td>L5</td><td>Time (min)</td><td></td><td></td></tr><tr><td>L6</td><td>Distance (km)</td><td></td><td></td></tr></table></td></tr>
</table>

	(d) Town A to town B *Press [2nd] [Y=] for [STAT PLOT],* *Choose **1: Plot 1*** *[ENTER] → **On** → [ENTER]* *Select **Type:** , [ENTER]* *Xlist: **L1**,* *Ylist: **L2***		
2	*(e) Town B to town C* *Press [2nd] [Y=] for [STAT PLOT], Choose **2: Plot 2** [ENTER]* *For On press [ENTER]* *Select **Type:** , [ENTER]* *Xlist: **L3**,* *Ylist: **L4***		*To change the **Xlist** ,* ***Press** [2nd] [3] for [L3] →[ENTER]* *To change the **Ylist**,* ***Press** [2nd] [4] for [L4]→[ENTER]*
	(f) Going back to town A *Press [2nd] [Y=] for [STAT PLOT], Choose **3: Plot 3** [ENTER]* *For On press [ENTER]* *Select **Type:** , [ENTER]* *Xlist: **L5**,* *Ylist: **L6***		*To change the **Xlist** ,* ***Press** [2nd] [5] for [L5] →[ENTER]* *To change the **Ylist**,* ***Press** [2nd] [6] for [L6]→[ENTER]*
3	*Press [WINDOW] and key in the setting.*		
4	*Press [GRAPH]*		*Speed-Time graph plotted.*

5	*Press* [*STAT*] *Select* ***CALC*** *Choose**4:LinReg(ax+b)*** [*ENTER*] *Press* [*2nd*][*1*]*for* [L1] [,] [*2nd*][*2*]*for* [L2][,] *Press* [*VARS*] *Select* ***Y-VARS*** *Choose**1:Function…*** [*ENTER*] *Choose**1:** Y_1 [*ENTER*] [*ENTER*]	     	

5	*Press [2nd][TRACE]* ***for*** *[CALC]* *Choose* ***7:*** $\int f(x)dx$ *[ENTER]* *Press [0] for lower limit, X = 0.* *[ENTER]* *Press [5] for upper limit, X = 5.* *[ENTER]*	    	*(a)* *The distance, in m, for the first 5 seconds* = ___ ***m***
6	*Press [STAT]* *Select* ***CALC*** *Choose****4:LinReg(ax+b)*** *[ENTER]* *Press [2nd][1]for [L1] [,]* *[2nd][2]for [L2][,]*		*From LinReg, the gradient, m is* ___

		LinReg y=ax+b a=3.2 b=0	*(b)* *The rate of change of speed, in m s^{-2}, in the first 5 seconds* = **m s^{-2}**
7	*Press* [STAT] *Select* ***CALC*** *Choose* ***4:LinReg(ax+b)*** [ENTER] *Press* [2nd] [5] *for* [L5] [,] [2nd] [6] *for* [L6] [ENTER]	y=ax+b a=3.2 b=0 LinReg(ax+b) L5, L6 LinReg y=ax+b a=⁻4 b=60	*From LinReg, the gradient, m is* *(c) The rate of change of speed, in ms^{-2}, in the last 4 seconds =*

Question for Discussion

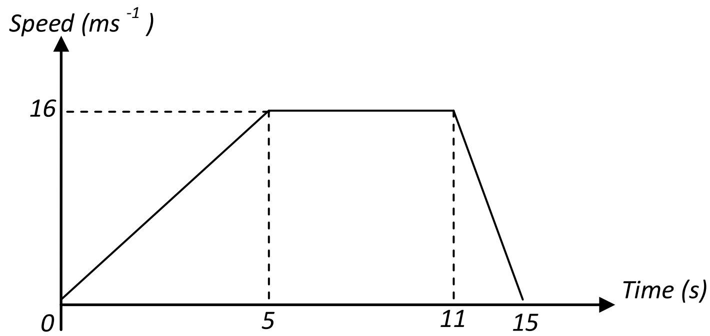
i. Can you guess the meaning of negative sign, ***a****?*

STUDENT'S WORKSHEET

Do the activity again without using graphing calculator.

ACTIVITY 2

Diagram shows speed-time graph of a particle for a period of 15 seconds.



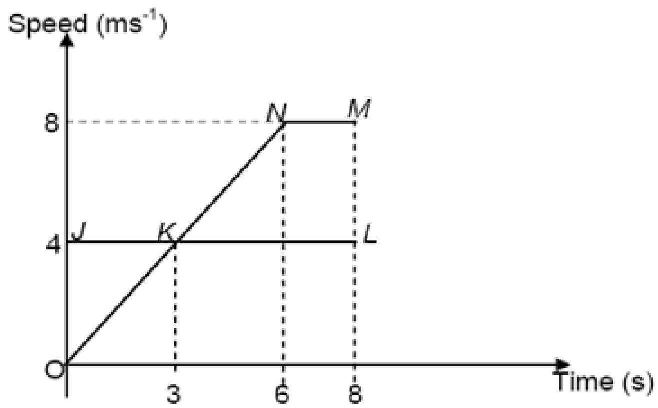
a) *Calculate the distance, in m, for the first 5 seconds.*
b) *Calculate the rate of change of speed, in $m\,s^{-2}$, in the first 5 seconds*
c) *Calculate the rate of change of speed, in ms^{-2}, in the last 4 seconds.*

ANSWER:

TEACHER'S NOTE

ENRICHMENT

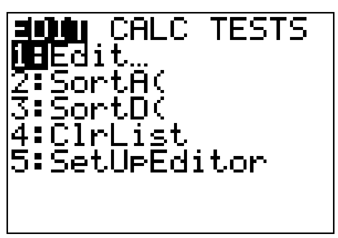
*Diagram shows the speed-time graph of two particles, **A** and **B** for a period of 8 seconds.*

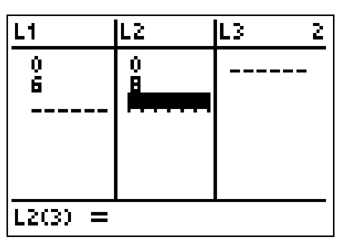


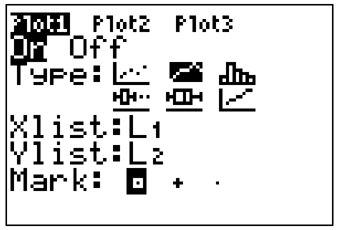
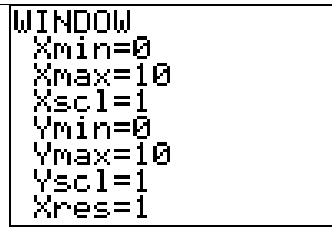
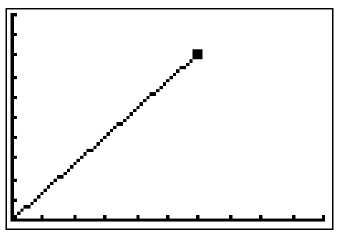
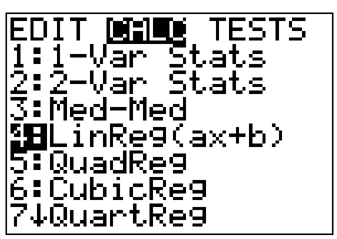
*The graph OKNM represents the movement of particle **A** and the graph JKL represents the movement of particle **B**. Both particles start moving at the same time.*

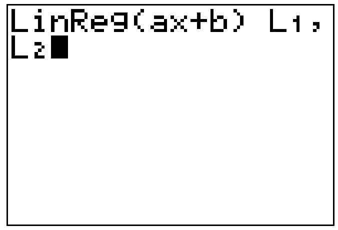
a) *Calculate the rate of change of speed, in ms^{-2}, of particle **A** in the first 6 s.*
b) *Calculate the rate of change of speed, in ms^{-2}, of particle **B** for a period of 8 s.*
c) *Find the distance, in m, when both particles meet.*

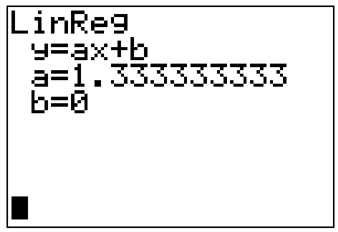
ANSWER:

<table>
<tr><th>Step</th><th>Procedure</th><th>Screenshot / key-stroke</th><th>Notes</th></tr>
<tr><td>1</td><td>Press STAT
Choose 1: Edit…</td><td>

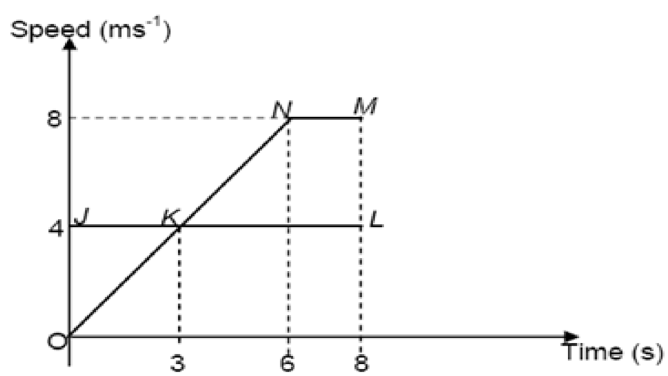

</td><td>Key in data as table below:<table><tr><th>L1 (Time)</th><th>L2 (Speed)</th></tr><tr><td>0</td><td>0</td></tr><tr><td>6</td><td>8</td></tr></table></td></tr>
</table>

2	*Press 2nd Y= for* *[STAT PLOT],* *Choose **1: Plot 1** ENTER* *For On press ENTER* *Select **Type:** , ENTER* *Xlist: **L1**,* *Ylist: **L2***		
3	*Press WINDOW and key in the setting.*		
4	*Press GRAPH*		*Speed-Time graph of **OKN** plotted.*
5	*Press STAT* *Select **CALC*** *Choose**4:LinReg(ax+b)*** *ENTER*	 	***LinReg (ax+b)** mode is an order to simulate the straight line equation, $y = mx + c$, where $m = a$, and $c = b$.*

<table>
<tr>
<td></td>
<td>Press [2nd][1] for [L1] [,]
[2nd][2] for [L2]
[ENTER]</td>
<td>

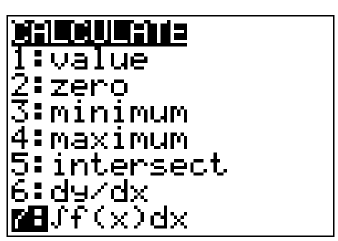
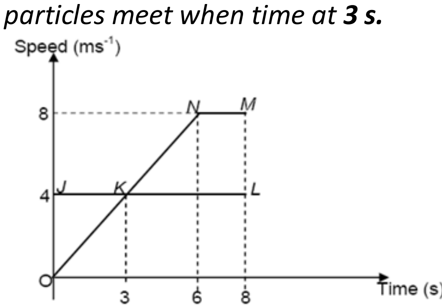

</td>
<td>From LinReg, the <u>gradient, m is</u>

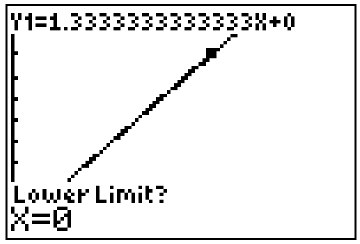
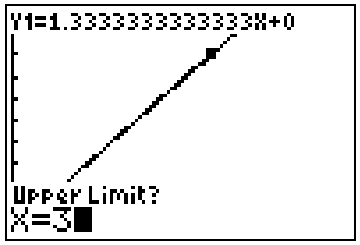
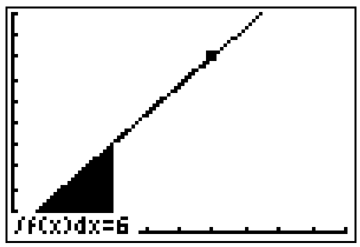
(a)
Calculate the <u>rate of change of speed</u>, in ms^{-2}, of particle A in the first 6 s.

= <strong><em>ms^{-2}</td>
</tr>
<tr>
<td colspan="4">

From the graph, obviously JKL is horizontal line; Therefore, the gradient m is 0.

(b)
Calculate the rate of change of speed, in ms^{-2}, of particle B in the 8 s.

= (uniform speed)</td>
</tr>
<tr>
<td>6</td>
<td>Press [2nd][TRACE]for [CALC]
Choose 7:$\int f(x)dx$
[ENTER]</td>
<td>
</td>
<td>From the graph, known that both particles meet when time at 3 s.

</td>
</tr>
</table>

	Press [0] for lower limit, X = 0. [ENTER]		
	Press [3] for upper limit, X = 3. [ENTER]		
			**(c)** **The distance, in m, when both particles meet.** **= [ ] m**

Questions for discussion

i. In certain cases, the area under a graph may not represent any meaningful quantity. Can you give one example?

ii. Can you find certain formulas for finding the area under a graph involving:

a. A straight line which is parallel to x-axis?

b. A straight line in the form of $y = kx + h$?

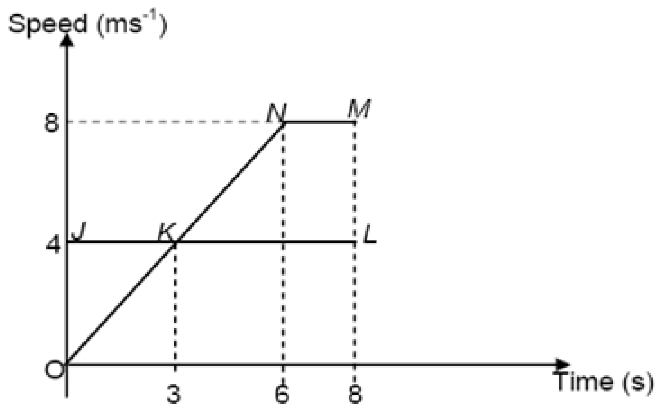
c. A combination of above?

STUDENT'S WORKSHEET

Do the activity again without using graphing calculator

ENRICHMENT

Diagram shows the speed-time graph of two particles, ***A*** *and* ***B*** *for a period of 8 seconds.*



The graph OKNM represents the movement of particle ***A*** *and the graph JKL represents the movement of particle* ***B****. Both particles start moving at the same time.*

a) *Calculate the rate of change of speed, in ms^{-2}, of particle* ***A*** *in the first 6 s.*
b) *Calculate the rate of change of speed, in ms^{-2}, of particle* ***B*** *for a period of 8 s.*
c) *Find the distance, in m, when both particles meet.*

ANSWER:

TOPIC **:** ***PROBABILITY 2***

LESSON OBJECTIVES **:**

Students will be able to…

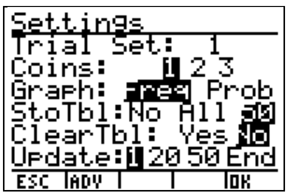
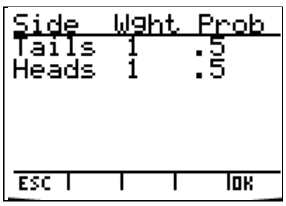
i. *Find the ratio of the number of times an event occurs to the number of trials*
ii. *Find the probability of an event from a big enough number of trials.*
iii. *Predict the occurrence of an outcome and make a decision based on known information.*

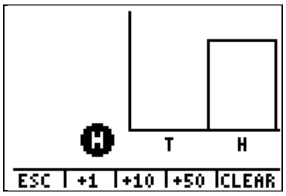
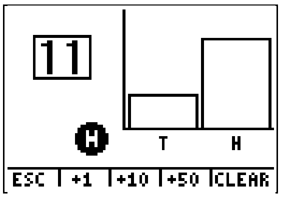
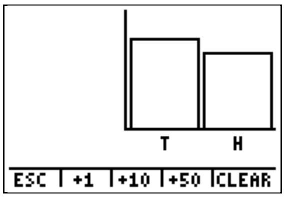
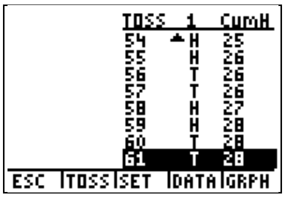
APPLICATION **:** *PROBABILITY SIMULATION*

NOTES **:**

> ***An observed probability*** *is based on data collected from experience or practical work such as flipping coins.*
>
> ***Theoretical probability is*** *the outcome from known quantities.*

Activity 1 : To apply probability concept from 'Tossing the Coins' simulation activity.

Step	Procedure	Screenshot / key-stroke	Notes
1	Press **APPS** choose **Prob Sim** ENTER ENTER **(or press any key)**		
2	Choose **1.Toss coins**		
3	Press **OK**. Press **ALPHA** **ZOOM** to get [F3]		Setting: to set number of trial. **Trial Set : 1** means the coin is toss once
4	Pres WINDOW **to get ADV**		To check that the chances of obtaining **HEAD** and **TAIL** is fair (Equal Probability) $\frac{1}{2} = 0.5$

Step	Procedure	Screenshot / key-stroke	Notes
5	Press GRAPH twice Press WINDOW to [**TOSS]** the coin and obtain either **HEAD** or **TAIL**.		
6	Press ZOOM for **[+10]** to toss 10 times.		The coin is tossed 10 times
7	Press TRACE for [+50]to toss 50 times		The coin is tossed 50 times
8	Press Y= to get **[ESC]** Press GRAPH to get Tabl to see the results in table form.		The table shows the number of tosses[TOSS], the result of the toss[1] and the cumulative HEAD tossed [CumH].

NOTE:

This ratio of **Head to Tosses** can be written as $\frac{number\ of\ HEAD}{total\ number\ of\ tosses}$ and is called the **Probability** of obtaining Head when randomly tossing a coin.

Discussion

*Can you figure events that produce **P(A) = 1** , and **P(A) = 0** ?*

TEACHER'S NOTE

Questions for discussion

i. Fill in your findings from the simulation,

The total number of Head	28
Total number of toss	61
Probability getting Head	$\frac{28}{61} = 0.46$

ii. Fill in the table with results from 5 other friends

	1	2	3	4	5	6
Probability	0.46					

iii. Compare and discuss their result

Similarity	Differences
The bigger the number of tosses the probability to be close to 0.5	Each person might not get the same ratio of head to tosses

iv. What can you conclude based on the results?

a) Knowledge about probability is very useful in making decision.

b) Prediction based on probability is not definite or absolute.

STUDENT'S WORKSHEET

Questions for discussion

i. **Fill in your findings from the simulation,**

The total number of Head	
Total number of toss	
Probability getting Head	

ii. **Fill in the table with results from 5 other friends**

	1	2	3	4	5	6
Probability						

iii. **Compare and discuss their result**

Similarity	**Differences**

iv. **What can you conclude based on the results?**

Activity 2

- Students can repeat the activity to compare their **theoretical probability** and their **observed probability**

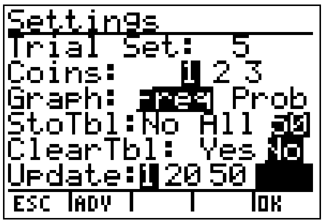
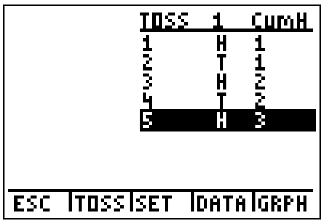
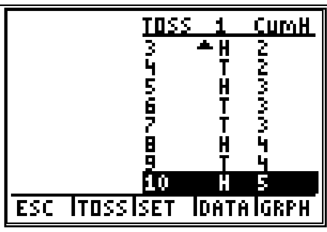
Steps	Procedure	Screenshot / key-stroke	Notes
1	In the **Settings,** Change **Trial Set:** to 5. Press GRAPH for **OK.**		The coin will be tossed 5 times for each set.
2	Start the activity: Press WINDOW to get **TOSS**, Press GRAPH to get **TABL** Transfer **CumH** from the calculator to **Table 1.** Find the probability of Head. Convert the probability in decimal form		
3	Repeat the step until 35 numbers of tosses.		

Table 1

Number Toss	Cumulative of heads [CumH]	Probability = cumulative /toss	In decimal form
5	3	3/5	0.6
10			
15			
20			
25			
30			
35			

TEACHER'S NOTE

DISCUSSION :

1. What do you notice about the probability of head as the number of tosses increases?

   The probability will be close to 0.5
   …………………………………………………………………………………………………….

2. What would be your theoretical probability of getting Head when you toss the coin 100 times?

   The probability will be close to 0.5
   ………

EXERCISES :

1. Suppose 250 people have applied for 18 job opening at a chain restaurant.

   i) What is the ratio of applicants will get the job to the number of applicants?

   $$\frac{18}{250} = 0.072$$

   ii) What is the probability of applicants will **not** be getting the job?

   $$1 - 0.072 = 0.928$$

2. Suppose there are **170** SPM leavers in your school. **52** of them have applied to be studying in private colleges. In a survey, **33** of them have will be studying in private colleges.

   i) What is the theoretical probability that the students will be studying in private colleges.

   $$\frac{52}{170} = 0.306$$

   ii) Based on your survey, what is the observed probability that they will be studying in private colleges.

   $$\frac{33}{170} = 0.194$$

3. The table shows the distribution of a group of 90 pupils playing a game.

	Form Four	Form Five
Girls	33	15
Boys	18	24

A pupil is chosen at random from the group to start the game.
What is the probability that **a girl** **from** **Form Four** will be chosen?

SPM 2005

$\frac{33}{90} = 0.367$

4. The table below shows how a group of **400** students travel to school.

Type of Transport	Bicycle	Motorcycle	Car	Bus
Number of students	100	70	80	150

A student is chosen from the group.
Find the probability that the student travels to school by car.

SPM 2009

$\frac{80}{400} = 0.2$

STUDENT'S WORKSHEET

DISCUSSION :

1. What do you notice about the probability of head as the number of tosses increases?

   ……

   ……

2. What would be your theoretical probability of getting Head when you toss the coin 100 times?

   ……

EXERCISES :

1. Suppose 250 people have applied for 18 job opening at a chain restaurant.

   i) What is the ratio of applicants will get the job to the number of applicants?

   ………

   ii) What is the probability of applicants will **<u>not</u>** be getting the job?

   ………

2. Suppose there are **170** SPM leavers in your school. **52** of them have applied to be studying in private colleges. In a survey, **33** of them have will be studying in private colleges.

   i) What is the theoretical probability that the students will be studying in private colleges.

   ………

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   ………

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………

SPM 2005

4. The table below shows how a group of 400 students travel to school.

Type of Transport	Bicycle	Motorcycle	Car	Bus
Number of students	100	70	80	150

A student is chosen from the group.
Find the probability that **the student** travels to school by **car**.

………

SPM 2009

TOPIC **:** ***FUNCTIONS***

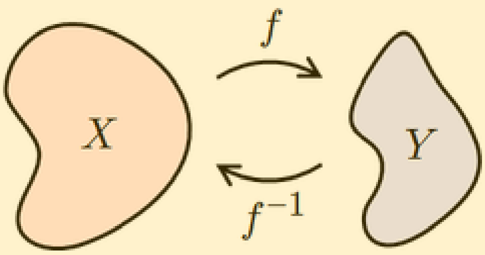
LESSON OBJECTIVE **:**

Students will be able to…

i. *find the object by inverse mapping given its image of function*
ii. *Use sketches to show the relationship between a function and its inverse*
iii. *Determine inverse functions using algebra*

NOTES **:**

In mathematics, if f is a function from a set A to a set B, then an inverse function for f is a function from B to A, with the property that a round trip (a composition) from A to B to A (or from B to A to B) returns each element of the initial set to itself. Thus, if an input x into the function f produces an output y, then inputting y into the inverse function f^{-1} (read f inverse)



$$\text{If } f(x) = y, \text{ then } f^{-1}(y) = x.$$

GRAPH OF INVERSE

This is identical to the equation $y = f(x)$ that defines the graph of f, except that the roles of x and y have been reversed. Thus the graph of f^{-1} can be obtained from the graph of f by switching the positions of the x and y axes. This is equivalent to the reflection of the graph across the line $y = x$.

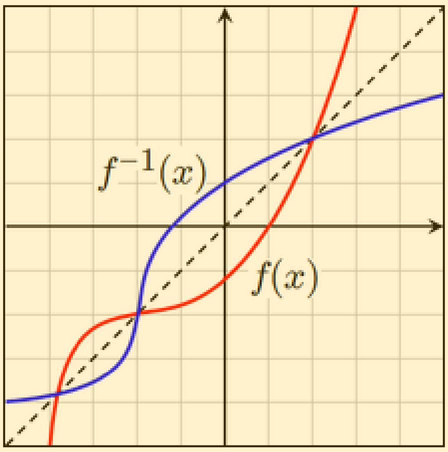


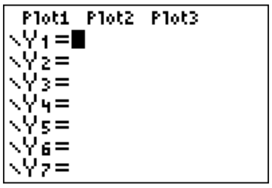
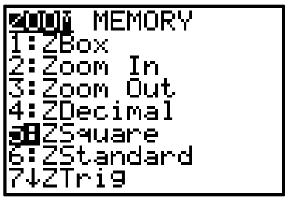
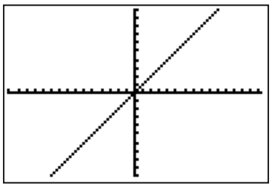
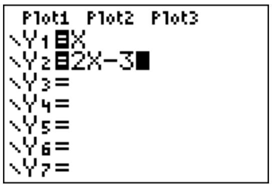
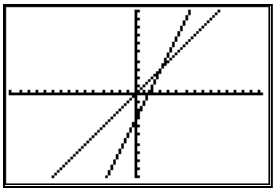
Diagram: The graphs of $y = f(x)$ and $y = f^{-1}(x)$ The dotted line is $y = x$

In this module, we will prove that the graph of *$y = f(x)$ and $y = f^{-1}(x)$ are reflecting one another across the line $y = x$.*

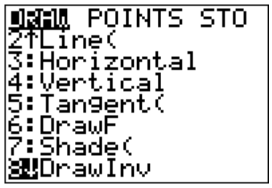
ACTIVITY 1 :

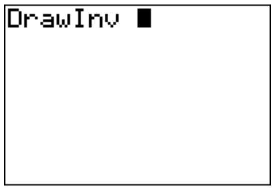
How to draw inverse functions graph?

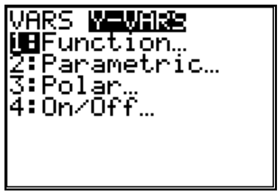
Eg: $y = 2x - 3$

Step	***Procedure***	***Screen***
1	*Press* [Y=] [X,T,Θ,n] *to graph the parent linear function.*	
2	Press [ZOOM] [5]	
Question for discussion	Why do we need the line, $y = x$?	
3	Press [Y=] [2] [X,T,Θ,n] [−] [3].	
4	Press [GRAPH]	

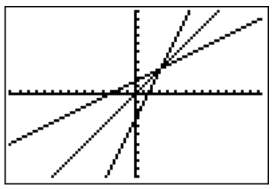
<table>
<tr><th>Step</th><th>Procedure</th><th>Screen</th></tr>
<tr><td>5</td><td>Next, direct the calculator to draw the inverse for $y=2x-3$

To access the DRAW menu, press [2nd][PRGM].
Select 8:DrawInv. This will paste the command to the home screen</td><td>

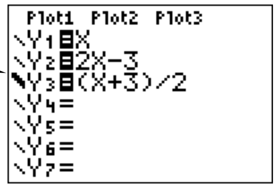

</td></tr>
<tr><td>6</td><td>Press [VARS], [▶] to Y-VARS.
Choose 1:Function… then choose 2:Y_2.

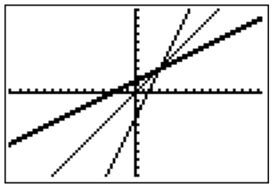
[ENTER]</td><td>


</td></tr>
<tr><td colspan="2">Question for discussion

Examine the inverse. What can you observe the pattern between $y=2x-3$ and its inverse?</td><td></td></tr>
</table>

<table>
<tr>
<td>7</td>
<td>Find the inverse of $y = 2x - 3$
By using algebra:-
Lets $x = 2y - 3$
$2y = x + 3$
$y = \dfrac{x+3}{2}$
Enter the inverse of $y = 2x - 3$ into Y₃.
Move the cursor to the left of Y₃

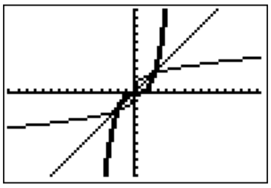
Press [ENTER] to change the line to a <u>thick line</u>.</td>
<td>
</td>
</tr>
<tr>
<td>8</td>
<td>Press [GRAPH].

What do you observe?</td>
<td></td>
</tr>
</table>

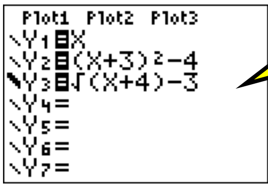
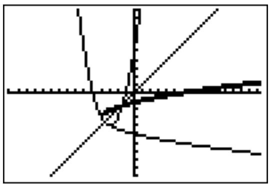
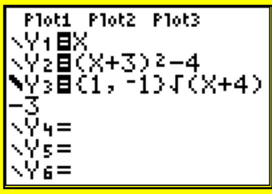
TEACHER'S NOTE

ACTIVITY 2 :

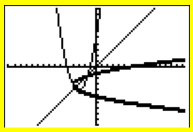
1. *Using the step above, find and draw graph of inverse function for* $y = \sqrt[3]{5x}$ **.**

Inverse function	*Graph*
Using algebra:- $y = \sqrt[3]{5x}$ Lets $x = \sqrt[3]{5y}$ $5y = x^3$ $y = \dfrac{x^3}{5}$	

2. *Find and draw graph of inverse function for* $y = (x+3)^2 - 4$

Inverse function	*Graph/ Screen*
Using algebra:- $y = (x+3)^2 - 4$ Lets $x = (y+3)^2 - 4$ $x = (y+3)^2 - 4$ $(y+3)^2 = x + 4$ $y + 3 = \sqrt{x+4}$ $y = \sqrt{x+4} - 3$	
*The equation in **Y₃** only accounts for half of the graph*	
*To graph the inverse you need **<u>both the positive and negative square root to achieve the proper result.</u>*** *Use the braces as shown (by pressing* [2nd][( ]*)*	

A student may come up with the result shown



STUDENT'S WORKSHEET

ACTIVITY 2 :

1. ***Using the step above, find and draw graph of inverse function for*** $y = \sqrt[3]{5x}$ **.**

Inverse function	***Graph***
Using algebra:-	

2. ***Find and draw graph of inverse function for*** $y = (x+3)^2 - 4$

Inverse function	***Graph***
Using algebra:-	

STUDENT'S WORKSHEET

ACTIVITY 3 :

From Activity 1 and Activity 2, you already learn how to find inverse of the function.
By using algebra, find the inverse function for the function below.

1	$y = 3x + 5$
2	$f(x) = \frac{x}{2} + 5$
3	$f(x) = \frac{3}{x-2}$
4	$f : x \to \frac{3x-1}{5}$
5	$f(x) = \frac{2x-7}{x+1}$

SPM QUESTIONS

SPM 2003 P1 Q2

Given that $g : x \rightarrow 5x+1$ *. Find* $g^{-1}(3)$

SPM 2004 P1 Q2

Given that the functions $h : x \rightarrow 4x+m$ *and* $h^{-1} : x \rightarrow 2kx + \frac{5}{8}$*, where m and k are constants, find the value of m and of k.*

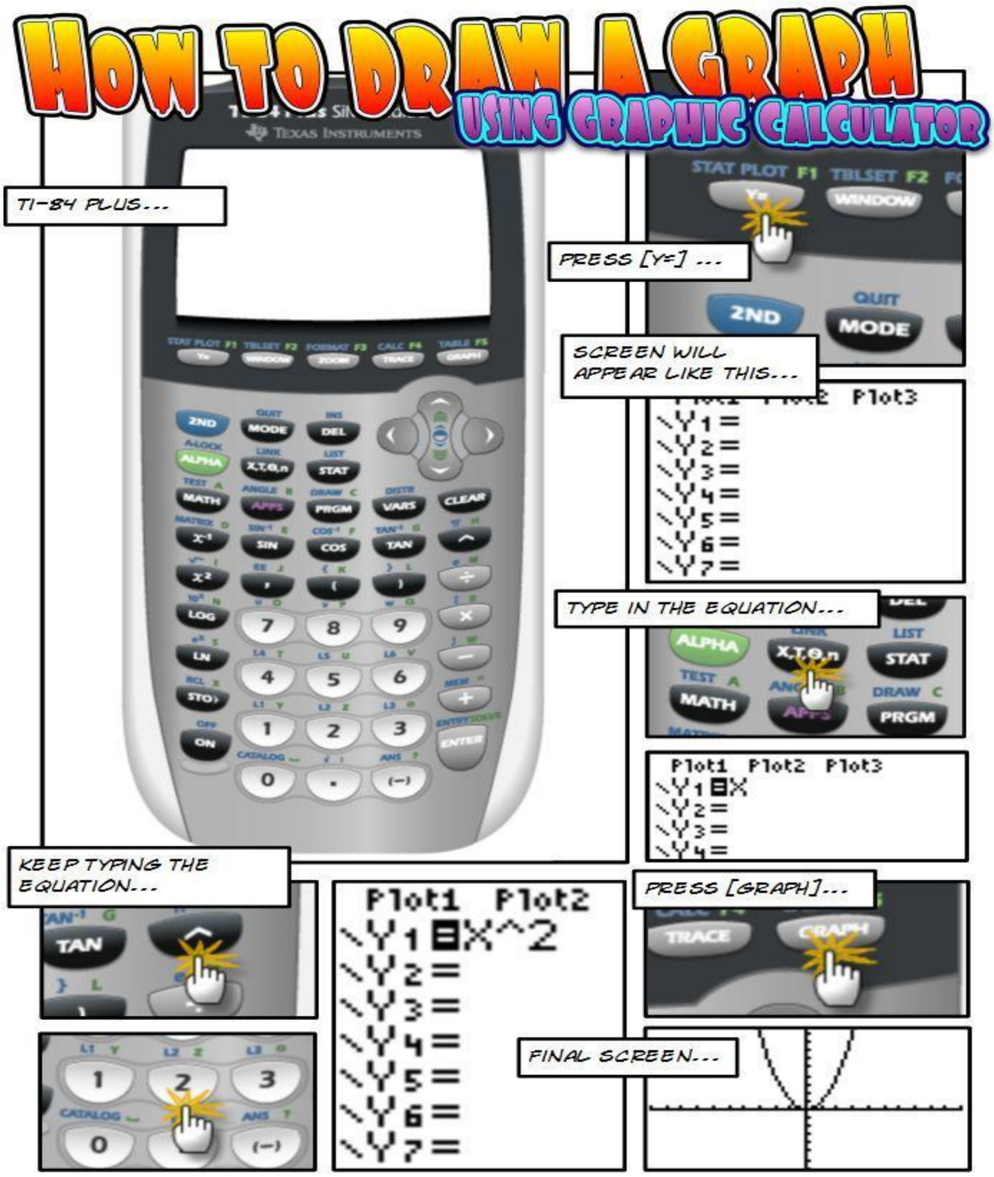
TOPIC : ***QUADRATIC FUNCTIONS***

SUB-TOPICS : *QUADRATIC FUNCTIONS AND GRAPHS*

LESSON OBJECTIVE :

Students will be able to…

i. *Recognize the shapes of graphs of function* $\boldsymbol{y = ax^2}$



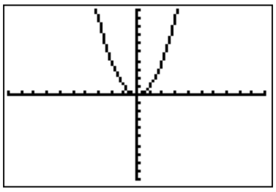
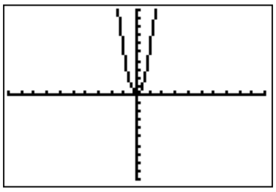
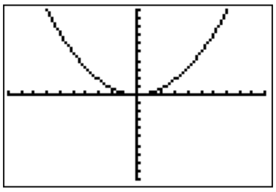
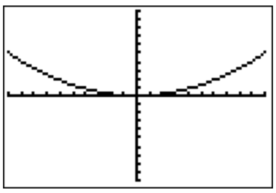
STEPS:

1) *Press* $\boxed{Y=}$
2) *Type the Function* $y = x^2$ → *press* $\boxed{X,T,\Theta,n}$, $\boxed{x^2}$, $\boxed{GRAPH}$, $\boxed{ZOOM}$ *press 4*
3) *Repeat step 1 & 2 for other quadratic functions.*
4) *To view graphs individually, press* $\boxed{Y=}$*, move cursor to the equal sign and press enter as shown in the diagram.*

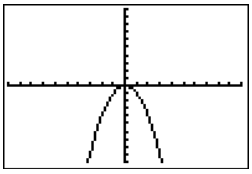
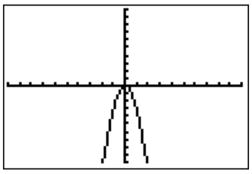
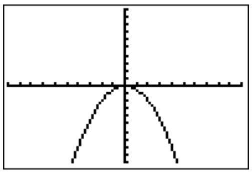
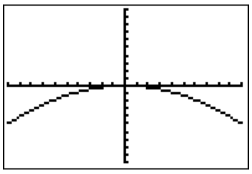
Y1 =■

TEACHER'S NOTE

COMPLETE THE TABLE BELOW USING GRAPHING CALCULATOR FOR THE VALUE OF $a > 0$ and $a < 0$

No	Function	Sketch	Value of a in $y = ax^2$	Does the parabola concave upward or downward?	Determine the turning point of the graph	Axis of symmetry	Describe the shape of the graph, (Standard, narrower or wider)
1	$y = x^2$		1	up	Minimum point	$x = 0$	Standard
2	$y = 5x^2$		5	up	Minimum point	$x = 0$	narrower
3	$y = 0.2x^2$		0.2	up	Minimum point	$x = 0$	wider
4	$y = 0.05x^2$		0.05	up	Maximum point	$x = 0$	wider

TEACHER'S NOTE

No	Function	Sketch	Value of *a* in $y = ax^2$	Does the parabola concave upward or downward?	Determine the turning point of the graph	Axis of symmetry	Describe the shape of the graph, (Standard, narrower or wider)
5	$y = -x^2$		-1	down	Maximum point	$x = 0$	Standard
6	$y = -3x^2$		-3	down	Maximum point	$x = 0$	narrower
7	$y = -0.5x^2$		−0.5	down	Maximum point	$x = 0$	wider
8	$y = -0.05x^2$		-0.05	down	Maximum point	$x = 0$	wider

TEACHER'S NOTE

Investigations

1. *Describe the effect on the graph* $y = \boldsymbol{a}x^2$ *as the value of* ***a*** *varies?*

   For $\boldsymbol{a} > 0$,
   - i. *The parabola* ***opens upwards***.
   - ii. *The vertex is* ***the lowest point***.
   - iii. *If the value of* ***a*** *is decreased, the shape of the graphs become* ***wider***.
   - iv. *If the value of* ***a*** *is increased, the shape of the graphs become* ***narrower***.

   For $\boldsymbol{a} < 0$,
   - i. *The parabola* ***opens downward.***
   - ii. *The vertex is* ***the highest point.***
   - iii. *If the value of* ***a*** *is decrease, the shape of the graphs become* ***wider.***
   - iv. *If the value of* ***a*** *is increase, the shape of the graphs become* ***narrower.***

2. *What happens when* ***a*** *approaches zero?*

   When a approaches zero, the graph appear to be a straight line.

3. *What happens when the value of* ***a*** *changes to negative?*

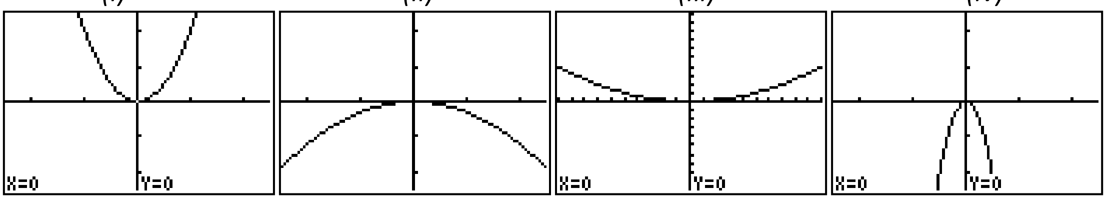
   When the values of a changes to negative, the straight line becomes a parabola again but concaves downward.

4. *Based on your finding, try to predict the shape of the graphs of the following equations:*

   i. $y = 2x^2$
   ii. $y = -0.3x^2$
   iii. $y = 0.04x^2$
   iv. $y = -8x^2$

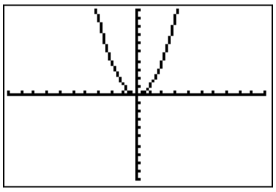
   Compare your answers with your friends.

(i) *(ii)* *(iii)* *(iv)*



STUDENT'S WORKSHEET

COMPLETE THE TABLE BELOW USING GRAPHING CALCULATOR

No	*Function*	*Sketch*	*Value of a in* $y = ax^2$	*Does the parabola concave upward or downward?*	*Determine the turning point of the graph*	*Axis of symmetry*	*Describe the shape of the graph, (Standard, narrower or wider)*
1	$y = x^2$		*1*	*up*	*Minimum point*	$x = 0$	*The same*
2	$y = 5x^2$						
3	$y = 0.2x^2$						*wider*
4	$y = 0.05x^2$				*Maximum point*		

STUDENT'S WORKSHEET

No	*Function*	*Sketch*	*Value of a in $y = ax^2$*	*Does the parabola concave upward or downward?*	*Determine the turning point of the graph*	*Axis of symmetry*	*Describe the shape of the graph, (Standard, narrower or wider)*
5	$y = -x^2$		-1				
6	$y = -3x^2$			*down*			
7	$y = -0.5x^2$						
8	$y = -0.05x^2$						

STUDENT'S WORKSHEET

Investigations

i. *Describe the effect on the graph $y = ax^2$ as the value of **a** varies?*

For $a > 0$,

- i. *The parabola……………………………………..*
- ii. *The vertex is ………………………………………*
- iii. *If the value of a is decreased, the shape of the graphs become ………………………*
- iv. *If the value of a is increased, the shape of the graphs become ……………………….*

For $a < 0$,

- i. *The parabola …………………………………*
- ii. *The vertex is …………………………………….*
- iii. *If the value of a is decrease, the shape of the graphs become ……………………………*
- iv. *If the value of a is increase, the shape of the graphs become …………………………….*

ii. *What happens when **a** approaches zero?*

……

iii. *What happens when the value of **a** changes to negative?*

……

iv. *Based on your finding, try to predict the shape of the graphs of the following equations:*

- i. $y = 2x^2$
- ii. $y = -0.3x^2$
- iii. $y = 0.04x^2$
- iv. $y = -8x^2$

Compare your answers with your friends.

TOPIC : *QUADRATIC FUNCTIONS*

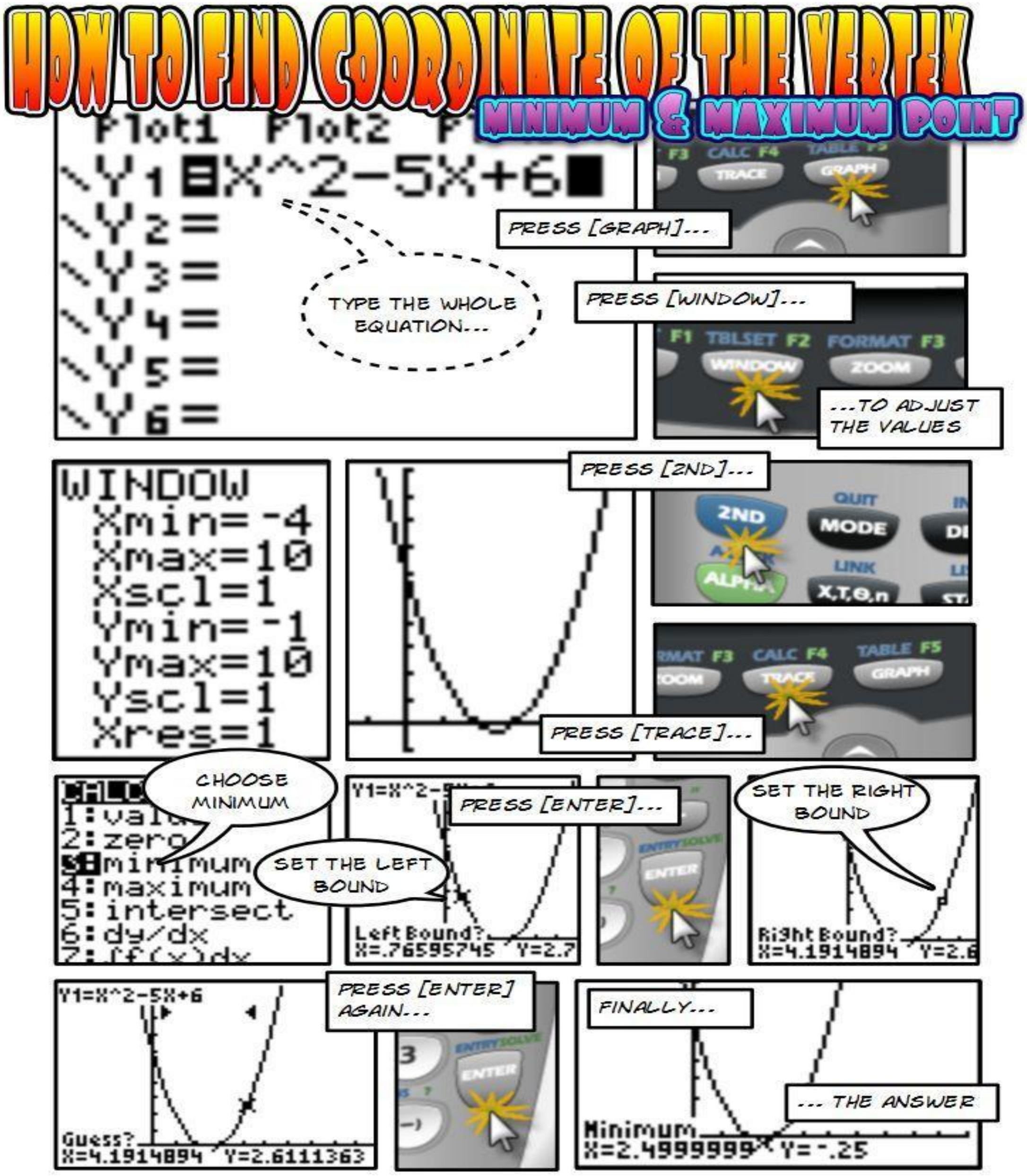
SUB-TOPICS : *QUADRATIC FUNCTIONS AND GRAPHS*

$$y = ax^2 + bx + c$$

LESSON OBJECTIVE :

Students will be able to…

i. *Recognize the shapes of graphs of functions*
ii. *Relate the position of quadratic functions graphs with types of roots for* $f(x) = 0$



TEACHER'S NOTE

USE A GRAPHING CALCULATOR TO DRAW A GRAPH OF EACH FUNCTION AND THEN COMPLETE THE TABLE BELOW FOR THE FUNCTION $\boldsymbol{y = ax^2 + bx + c}$

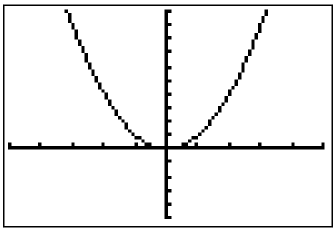
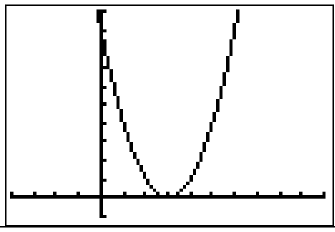
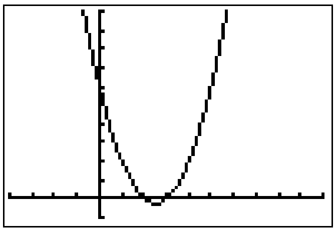
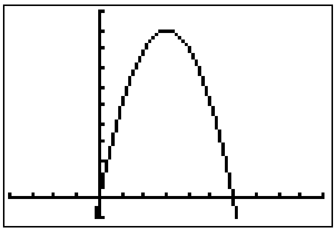
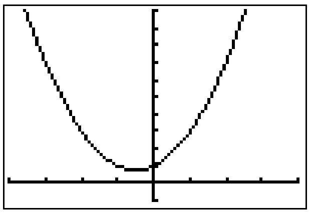
No	Function	Sketch	Value of			Does the parabola opens up or down	Coordinates of the vertex	Axis of symmetry
			a	*b*	*c*			
1	$y = x^2$		1	0	0	upwards	$(0,0)$	$x = 0$
2	$y = x^2 - 6x + 9$		1	-6	9	upwards	$(3,0)$	$x = 3$
3	$y = x^2 - 5x + 6$		1	-5	6	upwards	$\left(\frac{5}{2}, -\frac{1}{4}\right)$	$x = \frac{5}{2}$
4	$y = -x^2 + 6x$		-1	6	0	downwards	$(3,9)$	$x = 3$
5	$y = x^2 + x + 1$		1	1	1	upwards	$\left(-\frac{1}{2}, \frac{3}{4}\right)$	$x = -\frac{1}{2}$

Table 1

TEACHER'S NOTE

Investigations

1. *What do you notice about the axis of symmetry and the vertex of the graph?*

   The axis of symmetry passes through the vertex of the graph

2. *Based on your finding, complete the table below.*

Functions $y = f(x)$	Number of x-intercept of the graph	Value of $b^2 - 4ac$	Types of roots of the equation $f(x) = 0$
$y = x^2$	1	0	Equal roots
$y = x^2 - 6x + 9$	1	0	Equal roots
$y = x^2 - 5x + 6$	2	$1 > 0$	Two different roots
$y = -x^2 + 6x$	2	$40 > 0$	Two different roots
$y = x^2 + x + 1$	0	$-3 < 0$	No real root
$y = -x^2 - 4x - 5$	0	$-4 < 0$	No real root

Table 2

Hence, relate the position of quadratic function graphs with types of roots of the equation $f(x) = 0$.
If the roots are equal, the graph intercepts the x-axis.
If the roots are different, the graph intercepts the x-axis.
if there is no real root, there is no interception with the x-axis

3. *Based on the result in **Table 1**, express function $\boldsymbol{y = f(x)}$ in form $\boldsymbol{f(x) = a(x+p)^2 + q}$, where $\boldsymbol{p = -x}$, and $\boldsymbol{q = y}$ by looking at the **coordinates of the vertex** and state the **minimum/ maximum value***

Functions $y = f(x)$	Coordinates of the vertex	Functions $f(x) = a(x+p)^2 + q$	Minimum / Maximum value
$y = x^2 - 6x + 9$	$(3,0)$	$f(x) = (x-3)^2$	0
$y = x^2 - 5x + 6$	$\left(\frac{5}{2}, -\frac{1}{4}\right)$	$f(x) = \left(x - \frac{5}{2}\right)^2 - \frac{1}{4}$	$-\frac{1}{4}$
$y = -x^2 + 6x$	$(3,9)$	$f(x) = -(x-3)^2 + 9$	9
$y = x^2 + x + 1$	$\left(-\frac{1}{2}, \frac{3}{4}\right)$	$f(x) = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$	$\frac{3}{4}$
$y = -x^2 - 4x - 5$	$(2,-1)$	$f(x) = -(x-2)^2 - 1$	-1

Table 3

STUDENT'S WORKSHEET

USE A GRAPHING CALCULATOR TO DRAW A GRAPH OF EACH FUNCTION AND THEN COMPLETE THE TABLE BELOW FOR THE FUNCTION $y = ax^2 + bx + c$

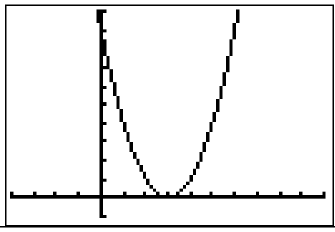
No	Function	Sketch	Value of			Does the parabola opens up or down	Coordinates of the vertex	Axis of symmetry
			a	*b*	*c*			
1	$y = x^2$							
2	$y = x^2 - 6x + 9$		1	-6	9	upwards	$(3,0)$	$x = 3$
3	$y = x^2 - 5x + 6$						$\left(\frac{5}{2}, -\frac{1}{4}\right)$	
4	$y = -x^2 + 6x$							
5	$y = x^2 + x + 1$							

Table 1

Investigations

1. *What do you notice about the axis of symmetry and the vertex of the graph?*

………

2. *Based on your finding, complete the table below.*

Functions $y = f(x)$	*Number of x-intercept of the graph*	*Value of* $b^2 - 4ac$	*Types of roots of the equation* $f(x) = 0$
$y = x^2$			*Equal roots*
$y = x^2 - 6x + 9$			
$y = x^2 - 5x + 6$		$1 > 0$	
$y = -x^2 + 6x$			*Two different roots*
$y = x^2 + x + 1$		$-3 < 0$	*No real roots*
$y = -x^2 - 4x - 5$			

Table 2

Hence, relate the position of quadratic function graphs with types of roots of the equation $f(x) = 0$.

If the roots are ………………………., the graph ………………………. the x-axis.
If the roots are ……………………., the graph ………………………. the x-axis.
If there is ………………………………., there is ……………………………. with the x-axis

3. *Based on the result in* ***Table 1****, express function* $\boldsymbol{y = f(x)}$ *in form* $\boldsymbol{f(x) = a(x+p)^2 + q}$ *where* $\boldsymbol{p = -x}$*, and* $\boldsymbol{q = y}$ *by looking at the* ***coordinates of the vertex*** *and state the* ***minimum/ maximum value***

Functions $y = f(x)$	*Coordinates of the vertex*	*Functions* $f(x) = a(x+p)^2 + q$	*Minimum / Maximum value*
$y = x^2 - 6x + 9$	$(3,0)$		
$y = x^2 - 5x + 6$			$-\frac{1}{4}$
$y = -x^2 + 6x$			
$y = x^2 + x + 1$		$f(x) = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$	
$y = -x^2 - 4x - 5$			

Table 3

TEACHER'S NOTE

PAST YEAR QUESTION (2007)

The quadratic function $f(x) = x^2 + 6x - 8$ *can be expressed in the form* $f(x) = (x+m)^2 - n$*, where m and n are constants. Find the values of m and n.*

Guide to use G.C to find answer.

1) *Type the function* $f(x) = x^2 + 6x - 8$
2) *Press* [GRAPH] *to view the function.*
3) *Press* [ZOOM]*, chose Z Standard,* [WINDOW]
4) *key in:*

```
WINDOW
 Xmin=-10
 Xmax=5
 Xscl=1
 Ymin=-20
 Ymax=10
 Yscl=1
 Xres=1
```

5) *Press* [GRAPH] *to view full graph.*
6) *Press* [TRACE]*,* [2nd]*, [CALC], enter → choose 3 minimum*
7) *Choose left bound* [ENTER]*, right bound* [ENTER]*,* [ENTER]*.*

Answer
1) Minimum point (-3, -17), Hence $m = 3, n = 17$

Clone SPM 2007

The quadratic function $f(x) = 4x^2 - 16x + 8$ *can be expressed in the form* $f(x) = a(x+p)^2 + q$*, where a, p and q are constants.*

a) *Determine the values of a, p and q.*
b) *State the axis of symmetry and the coordinates of the minimum point of the graph of* $f(x)$*.*

Answer

a) $a = 4, p = -2, q = -8$
b) $x = 2$, *Minimum Point* $(2, -8)$

Hint :

```
WINDOW
 Xmin=-2
 Xmax=5
 Xscl=1
 Ymin=-12
 Ymax=12
 Yscl=1
 Xres=1
```

STUDENT'S WORKSHEET

PAST YEAR QUESTION (2007)

The quadratic function $f(x) = x^2 + 6x - 8$ *can be expressed in the form* $f(x) = (x+m)^2 - n$*, where m and n are constants. Find the values of m and n.*

Answer

Hint:

```
WINDOW
 Xmin=-10
 Xmax=5
 Xscl=1
 Ymin=-20
 Ymax=10
 Yscl=1
 Xres=1
```

CLONE SPM 2007

The quadratic function $f(x) = 4x^2 - 16x + 8$ *can be expressed in the form* $f(x) = a(x+p)^2 + q$*, where a, p and q are constants.*

c) *Determine the values of a, p and q.*
d) *State the axis of symmetry and the coordinates of the minimum point of the graph of* $f(x)$.

Answer

Hint :

```
WINDOW
 Xmin=-2
 Xmax=5
 Xscl=1
 Ymin=-12
 Ymax=12
 Yscl=1
 Xres=1
```

TOPIC : ***SIMULTANEOUS EQUATIONS***

SUB TOPIC : *ONE LINEAR EQUATION AND ONE NON-LINEAR EQUATION*

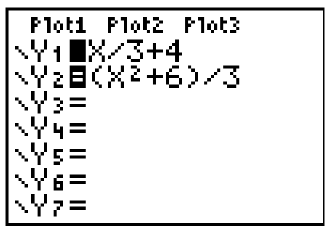
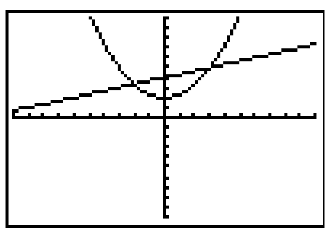
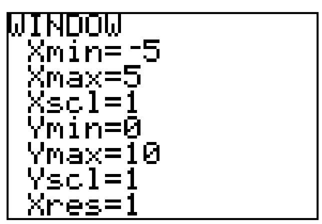
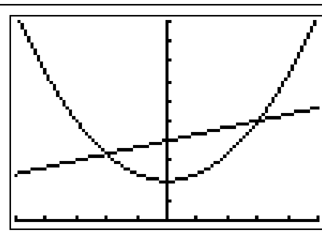
LESSON OBJECTIVE :

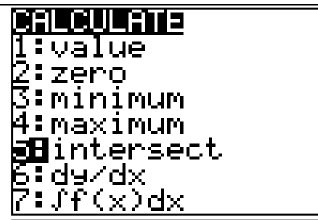
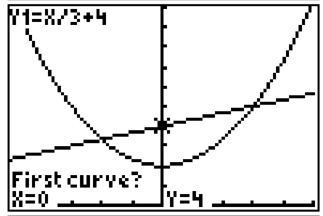
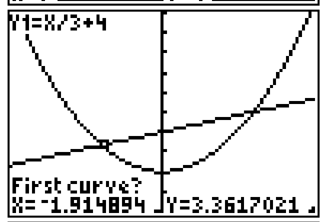
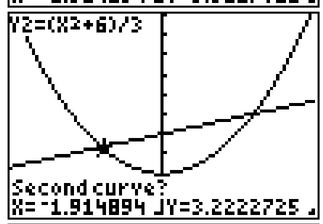
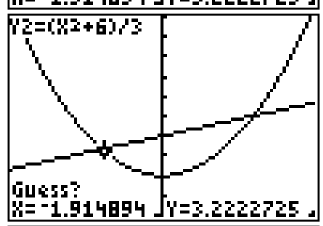
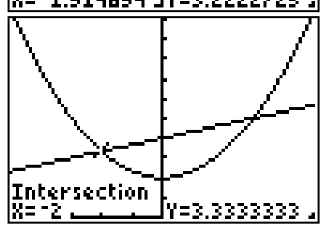
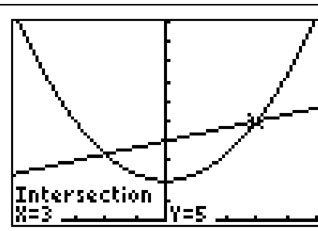
Students will be able to…

i. to solve one linear equation and one non-linear equation simultaneously

EXAMPLE QUESTION : ***Solve the simultaneous equations***

$$y = \frac{1}{3}x + 4 \textbf{ and } 3y - x^2 - 6 = 0$$

Step	***Procedure***	***Screenshot / key-stroke***	***Notes***
1	*Press* [Y=] *then* [X,T,Θ,n] [÷] [3] [+] [4] *Move the cursor to* Y_2, *then press* [(] [X,T,Θ,n] [x^2] [+] [6] [)] [÷] [3]		*In order to solve the simultaneous equation, make* ***y as the subject*** *of both equations.* $y = \frac{1}{3}x + 4$, **and** $y = \frac{x^2 + 6}{3}$
2	*Press* [GRAPH] *Press* [ZOOM] *choose* ***6: Z Standard*** *press* [ENTER]		
3	***TO VIEW THE FULL GRAPH*** *Press* [WINDOW] *change the* ***setting***		
4	*Press* [GRAPH]		

5	***TO SEE THE INTERSECTION POINTS,*** *Press* [2nd][TRACE] ***for*** [CALC] ***mode*** *choose **5: Intersect*** *press*[ENTER] *Use the cursor to **move the point nearby** one of the intersection point.* *Accept the First line /curve by pressing*[ENTER]. *Accept the Second line / curve by pressing*[ENTER]. *Guess a value of x or press* [ENTER] *again.* *Read the coordinate of the intersection*	     	*What is the value of the first intersection?* $x =$ ☐ ***and*** $y =$ ☐
6	***Repeat step 5** to obtain the second intersection point* *Read the coordinate of the intersection*		*What is the value of the second intersection point?* $x =$ ☐ ***and*** $y =$ ☐

DISCUSSION:-

1) *What can you say about the graphs?*

   i. *The linear function makes a_____________________, while the quadratic function makes a _____________________*

   ii. *The two graphs ____________________at two points.*

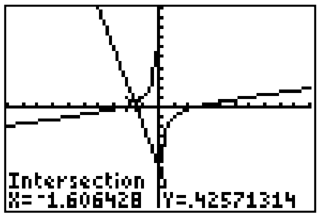
2) *What are the solutions of the simultaneous equations?*

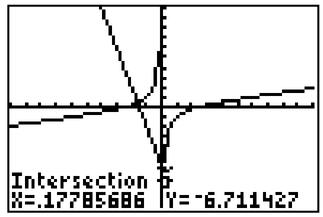
   i. *The _______________________ of the two graphs are the solutions of the simultaneous equations, which are (___ , ___) and (___ , ___).*

TEACHER'S NOTE

1) *SOLVE THE SIMULTANEOUS EQUATION GIVEN USING GRAPHING CALCULATOR*

$$4x + y = -6 \ \ and\, x^2 - 5xy = 6$$





DISCUSSION:-

a) *What can you say about the graphs?*

   i. *A straight line and a reciprocal graph.*
   ii. *The linear function makes a straight line, while the reciprocal function makes a reciprocal graph*
   iii. *The two graphs intersect at two points.*

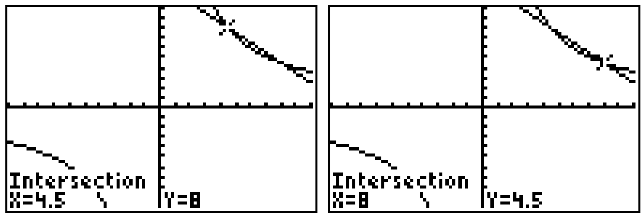
b) *What are the solutions of the simultaneous equations?*

   The intersections of the two graphs are the solution of the simultaneous equations, which is $(\mathbf{0.18}, \mathbf{-6.71})$ *and* $(\mathbf{1.6}, \mathbf{0.4})$.

TEACHER'S NOTE

2) *SOLVE THE SIMULTANEOUS EQUATION GIVEN USING GRAPHING CALCULATOR*

$$xy = 36 \text{ } and \text{ } 2x + 2y = 25$$



INVESTIGATION:-

a) *What can you say about the graphs?*

   i. *A straight line and a reciprocal graph.*
   ii. *The linear function makes a straight line, while the reciprocal function makes a reciprocal graph*
   iii. *The two graphs intersect at two points.*

b) *What are the solutions of the simultaneous equations?*

The intersections of the two graphs are the solution of the simultaneous equations, which is $(\mathbf{4.5}, \mathbf{8})$ *and* $(\mathbf{8}, \mathbf{4.5})$.

3) *SOLVE THE SIMULTANEOUS EQUATION GIVEN USING GRAPHING CALCULATOR*

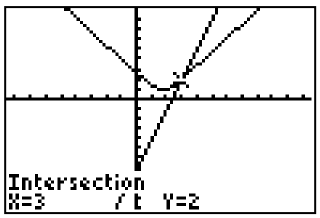
$$3x - y = 7 \ and \, x^2 - xy + y^2 = 7$$

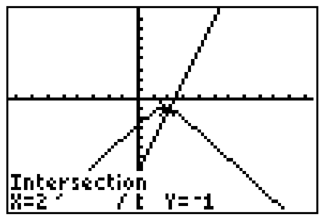
The equations can be simplified like this:

$y = 3x - 7$

$y = +\sqrt{7 - x^2 + x(3x - 7)}$

$y = -\sqrt{7 - x^2 + x(3x - 7)}$





INVESTIGATION:-

a) *What can you say about the graphs?*

i. *A straight line and a quadratic graph.*
ii. *The linear function makes a straight line, while the quadratic function makes a quadratic graph/ curve*
iii. *The two graphs intersect at one point.*
iv. *Since the quadratic function has square root to be translated as positive and negative values, the functions also varies.*

b) *What are the solutions of the simultaneous equations?*

The intersections of the two graphs are the solution of the simultaneous equations, which is $(3, 2)$ *and* $(2, -1)$.

STUDENT'S WORKSHEET

1) *SOLVE THE SIMULTANEOUS EQUATION GIVEN USING GRAPHING CALCULATOR*

$$4x + y = -6 \;\; and\, x^2 - 5xy = 6$$

Intersection 1	***Intersection 2***

INVESTIGATION:-

a) *What can you say about the graphs?*

i. *A ………………………………… and a ………………………… graph.*

ii. *The linear function makes a ……………………………………………………………………*
while the reciprocal function makes a ………………………………………………………

iii. *The two graphs ……………………………………… at two points.*

b) *What are the solutions of the simultaneous equations?*

The ……………………………………… of the two graphs are the solution of the simultaneous equations, which are (,) and (,).

STUDENTS' WORKSHEET

2) *SOLVE THE SIMULTANEOUS EQUATION GIVEN USING GRAPHING CALCULATOR*

$$xy = 36 \text{ and } 2x + 2y = 25$$

Intersection 1	***Intersection 2***

INVESTIGATION:-

a) *What can you say about the graphs?*

  i. *A ………………………………….. and a ………………………….. graph.*
  ii. *The linear function makes a …………………………………………………………………………….. while the reciprocal function makes a …………………………………………………………….*
  iii. *The two graphs ………………………………………. at two points.*

b) *What are the solutions of the simultaneous equations?*

The ……………………………………….. of the two graphs are the solution of the simultaneous equations, which are (,) and (,)

STUDENT'S WORKSHEET

3) *SOLVE THE SIMULTANEOUS EQUATION GIVEN USING GRAPHING CALCULATOR*

$$\mathbf{3x - y = 7 \ and \ x^2 - xy + y^2 = 7}$$

Hint: - simplified the quadratic equation:

$y = 3x - 7$

$y = +\sqrt{7 - x^2 + x(3x - 7)}$

$y = -\sqrt{7 - x^2 + x(3x - 7)}$

Intersection 1	***Intersection 2***

INVESTIGATION:-

a) *What can you say about the graphs?*

i. *A .... and a....................................... graph.*
ii. *The linear function makes a ...., while the quadratic function makes a ....*
iii. *The graphs intersect at .... point.*
iv. *Since the quadratic function has square root to be translated as .... and .... values, the functions also varies.*

b) *What are the solutions of the simultaneous equations?*

The <u>intersections</u> of the two graphs are the solution of the simultaneous equations, which are

(,) *and* (,).

TEACHER'S NOTE

PAST YEAR QUESTION (2003)

Solve the simultaneous equations $4x + y = 2$ *and* $x^2 + x - y = 12$.

Solution by using G.C

1. In order to solve this question, make y *as the subject of both equations as following:*

$$y = 2 - 4x$$

$$y = x^2 + x - 12$$

2. Follow step 1 and 2 as provided for the example question.

3. For step 3, change the setting for $\boxed{\text{WINDOW}}$ *as shown.*

4. Follow steps 4 to 6 as provided for the example question.

```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=1
 Ymin=-15
 Ymax=35
 Yscl=1
 Xres=1
```

Answer

$x = -7,\ y = 30$

$x = 2, y = -6$

SPM CLONE (2007)

Solve the following simultaneous equations,

$$2x - y - 11 = 0$$

$$2x^2 - 10x + y + 17 = 0$$

Answer

$x = 1, y = -9$

$x = 3, y = -5$

Hint:

```
WINDOW
 Xmin=-5
 Xmax=10
 Xscl=1
 Ymin=-15
 Ymax=5
 Yscl=1
 Xres=1
```

STUDENT'S WORKSHEET

PAST YEAR QUESTION (2003)

Solve the simultaneous equations $4x + y = 2$ *and* $x^2 + x - y = 12$.

Answer

SPM CLONE (2007)

Solve the following simultaneous equations,

$$2x - y - 11 = 0$$
$$2x^2 - 10x + y + 17 = 0$$

Answer

TOPIC **:** ***COORDINATE GEOMETRY***

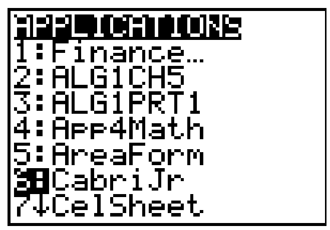
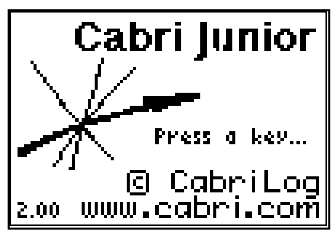
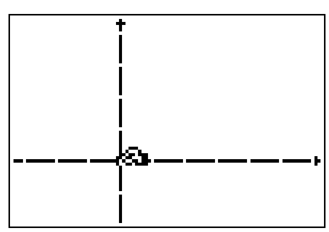
SUB TOPIC : *PERPENDICULAR LINES*

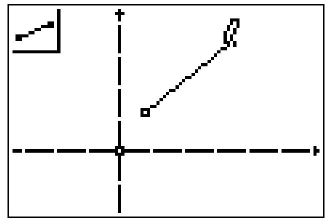
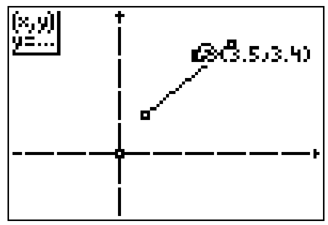
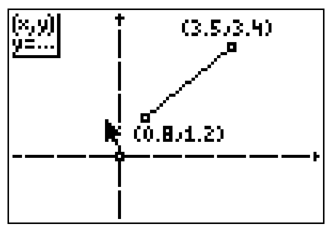
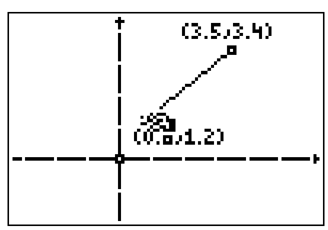
LESSON OBJECTIVE **:**

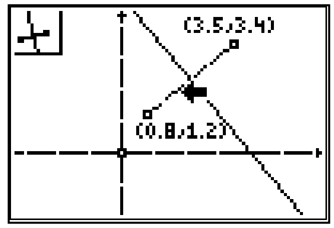
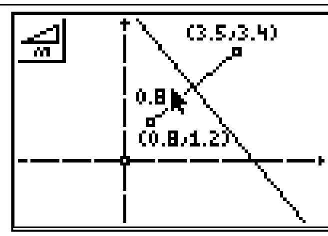
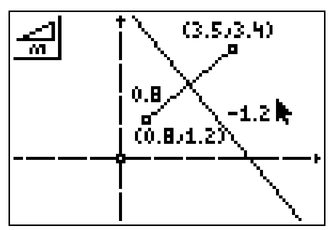
Students will be able to…

i. to determine the relationship between the gradients of perpendicular lines

APPLICATIONS **:** *CABRI JR*

Step	***Procedure***	***Screenshot / key-stroke***	***Notes***
1	***<u>Accessing Cabri Jr</u>*** *Press [APPS] and select **Cabri Jr*** *press [ENTER]* *Press any key to proceed*	 	
2	***<u>DRAWING THE LINE SEGMENT</u>*** *Press [GRAPH] to access [F5] menu* *Select **Hide/Show*** *Press [▶], Select **Axes*** *press [ENTER]* *To move the axes, press [◀] , [▼] to the origin.* *Press [ALPHA].* *A hand cursor will appear.* *By using [▲], [▼], [◀] [▶] to fix the position of the axes.* *Press [ENTER].*		

3	*Go to* WINDOW *menu and select* ***Segment*** *Press* ENTER *A* ***small box on the top left*** *indicate the* ***active menu*** *button.* *Move the pencil anywhere on the screen to plot the first point and press* ENTER *Press* ▶ **,** ▲ ***to*** *drag the segment* ENTER		
4	***DISPLAY COORDINATES*** *Press* GRAPH *and select* ***Coord. & Eq.*** *Press* ENTER *Move the cursor to highlight a point until the pointer blinking.* *Press* ENTER **.** ENTER**.** *Move the cursor to* ***highlight the second point*** *and press* ENTER **.** ENTER **.** *Press* CLEAR *to* ***exit*** *the active menu button*	 	
5	***TO MOVE THE POINTS*** *Move cursor to highlight a point and press* ALPHA *and a* ***hand cursor*** *will appear.* ***Move the point*** *to the desired position and press* ENTER		

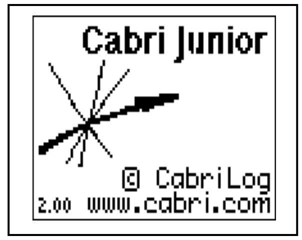
6	***CONSTRUCT PERPENDICULAR LINE*** *Press* ZOOM *menu and select* ***Perp.*** *to construct a perpendicular line.* *Press* ENTER. *To* ***position the perpendicular line,*** *move the cursor* ***on the line segment.*** *Press* ENTER***.*** *A perpendicular line will appear.* *To fix the perpendicular line, press* ENTER *Press* CLEAR *to* ***exit the active menu*** *button*		
7	***Measuring Slope*** *Press* GRAPH. *Select* ***Measure.*** *Press* ▶ *and choose* ***Slope****.* *Press* ENTER. *Move the cursor to* ***highlight the line segment*** *and press* ENTER***.*** ENTER *Move the cursor to* ***highlight the perpendicular line*** *and press* ENTER***.*** ENTER		*Gradient of the line segment is 0.8 (appear on the screen)* *Gradient of the perpendicular line segment is – 1.2 (appear on the screen)*

TEACHER'S NOTE

INSTRUCTIONS:

Steps to use the Ti- Graphing calculator to investigate $m_1 m_2 = -1$

- *Press* APPS *and choose the* ***CabriJr*** *Application.*
- *Press any key*
- *Follow step 1 – 7*
- *Complete the table below*



INVESTIGATION

Coordinates for the line segment	***Gradient of the line segment,*** m_1	***Gradient of the perpendicular line,*** m_2 *[To* ***position the perpendicular line,*** *move the cursor* ***on the line segment.*** *Press* ZOOM *menu and select* ***Perp.*** *to construct a perpendicular line. Press* ENTER*.]*	***Product of*** m_1 ***and*** m_2
(-0.6, 3) and (-3, -2)	*2.0*	*-0.5*	*-1*
(-2.5, 3) and (1.5, -1.1)	*-1*	*1*	*-1*
(-3, -1.5) and (2, 1)	*0.5*	*-2*	*-1*

1. *What do you notice about the perpendicular line when you move any points on the line segment?*
   The perpendicular line moves according to the new line segment.

2. *What is the relationship between the gradient of the line segment and its perpendicular line?*
   The product of the gradient of the line segment and it's perpendicular line is -1

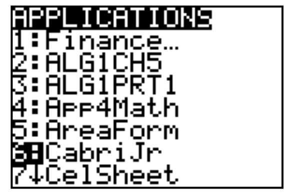
3. *Write the relationship between the gradient of the line segment and its perpendicular line in mathematical term.*
   $m_1 \times m_2 = -1$

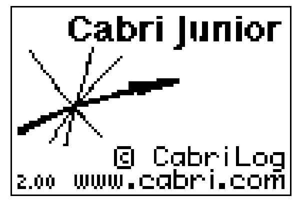
STUDENT'S WORKSHEET

INSTRUCTIONS:

Steps to use the TI- Graphing calculator to investigate $\boldsymbol{m_1 m_2 = -1}$

- *Press* [*APPS*] *and choose the* ***CabriJr*** *Application.*
- *Press any key*
- *Follow* ***step 1 – 7***
- *Complete the table below*





INVESTIGATION

Coordinates for the line segment	***Gradient of the line segment,*** $\boldsymbol{m_1}$	***Gradient of the perpendicular line,*** $\boldsymbol{m_2}$ *[To* ***position the perpendicular line,*** *move the cursor* ***on the line segment.*** *Press* [*ZOOM*] *menu and select* ***Perp.*** *to construct a perpendicular line.* *Press* [*ENTER*]*.]*	***Product of*** $\boldsymbol{m_1}$ ***and*** $\boldsymbol{m_2}$
(-0.6, 3) and (-3, -2)			
(-2.5, 3) and (1.5, -1.1)			
(-3, -1.5) and (2, 1)			

1. *What do you notice about the perpendicular line when you move any points on the line segment?*

   ……

2. *What is the relationship between the gradient of the line segment and its perpendicular line?*

   ……

3. *Write the relationship between the gradient of the line segment and its perpendicular line in mathematical term.*

   ……

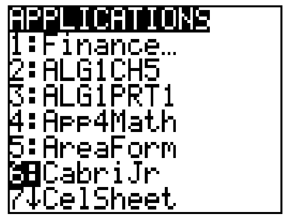
TEACHER'S NOTE

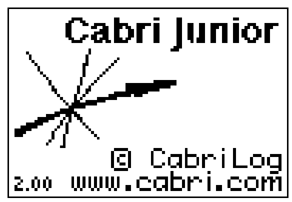
INVESTIGATION

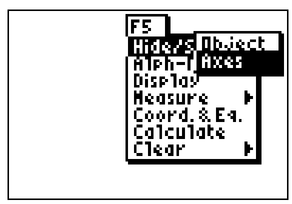
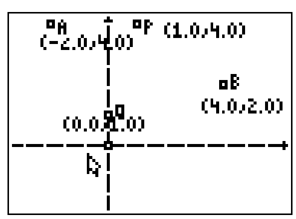
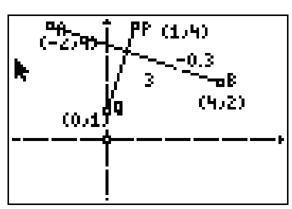
Given the points A (-2,4) , B (4,2) , P(1,4) and Q(0,1). Using CabriJr application, show that AB is perpendicular to PQ

<table>
<tr><th>Procedure</th><th>Screenshot / key-stroke</th><th>Notes</th></tr>
<tr><td>INSTRUCTIONS:

Steps to use the Ti- Graphing calculator to investigate $m_1m_2 = -1$

1. Press [APPS] and choose the CabriJr Application.
2. Press any key</td><td>

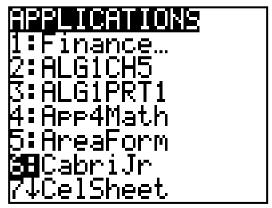
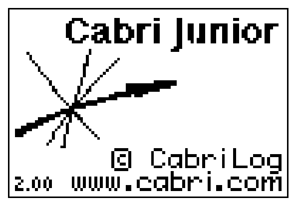

</td><td></td></tr>
<tr><td>3. Press [GRAPH]

4. Choose axes</td><td>
</td><td></td></tr>
<tr><td>5. Adjust the axes and plot all the points given</td><td>
</td><td></td></tr>
<tr><td>6. Construct a segment between AB and PQ measure the slope.
(To plot the points, refer the procedure above)</td><td>
</td><td></td></tr>
</table>

STUDENT'S WORKSHEET

INVESTIGATION

Given the points A (-2,4) , B (4,2) , P(1,4) and Q(0,1). Using CabriJr application, show that AB is perpendicular to PQ

Procedure	***Screenshot / key-stroke***	***Notes***
Steps to use the Ti- Graphing calculator to investigate $m_1m_2 = -1$ 1. *Press* $\boxed{APPS}$ *and choose the* ***CabriJr*** *Application.* 2. *Press any key*	 	

TOPIC : ***DIFFERENTIATIONS***

LESSON OBJECTIVES :

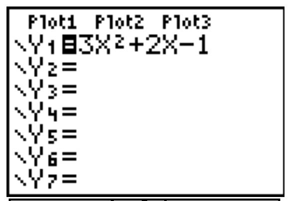
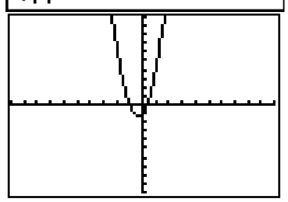
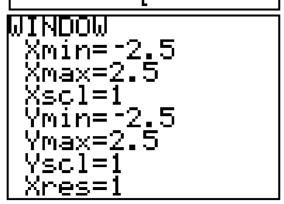
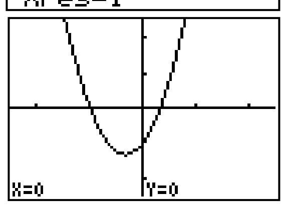
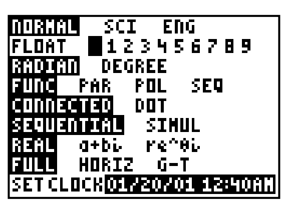
Students will be able to…

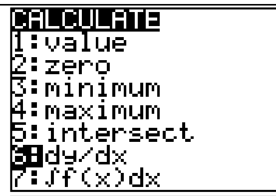
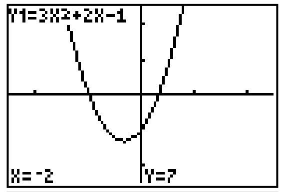
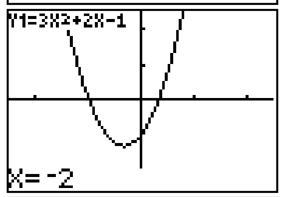
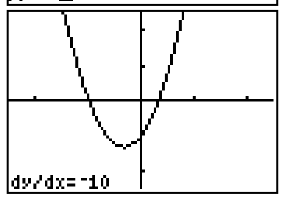
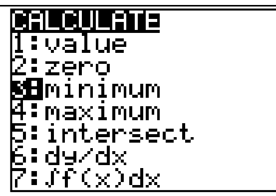
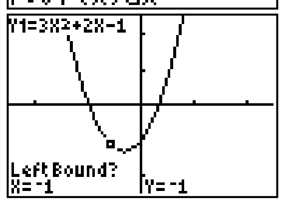
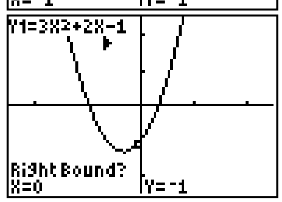
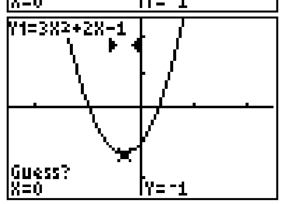
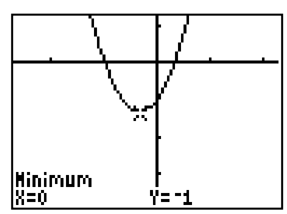
i. *Understand and use the concept of maximum and minimum values to solve problems.*
ii. *Determine the gradient of tangent at a point on a curve.*
iii. *Find maximum or minimum values.*

EXAMPLE : *Draw the graph* $y = 3x^2 + 2x - 1$ *, find*

a) $\frac{dy}{dx}$ *When x = -2*

b) Maximum or minimum values

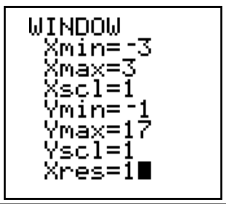
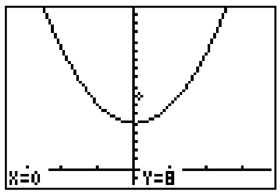
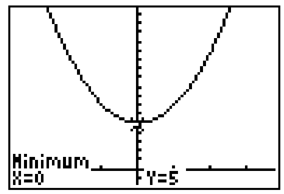
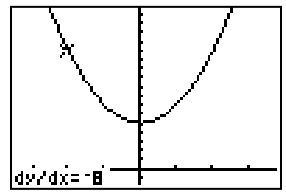
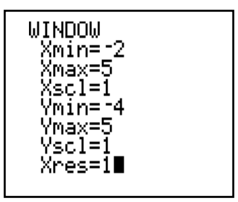
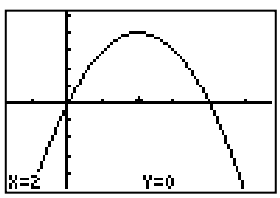
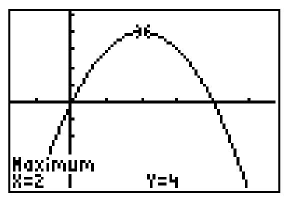
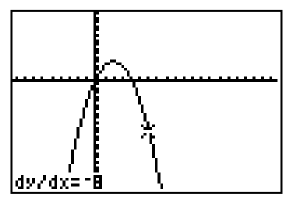
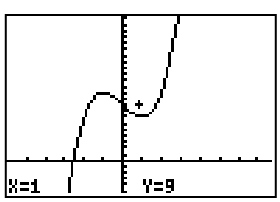
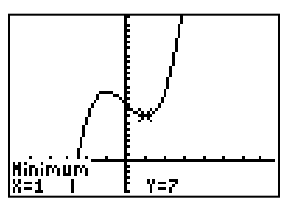
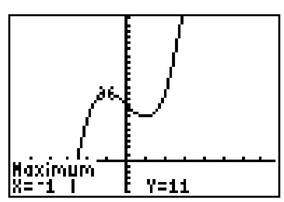
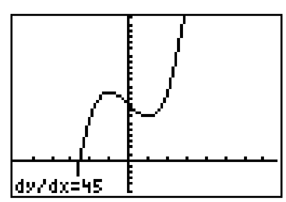
Step	***Procedure***	***Screenshot / key-stroke***	***Notes***
1	***Draw the graph*** $y = 3x^2 + 2x - 1$ *Press* $\boxed{Y=}$ $\boxed{3}$ $\boxed{X,T,\Theta,n}$ $\boxed{x^2}$ $\boxed{+}$ $\boxed{2}$ $\boxed{X,T,\Theta,n}$ $\boxed{-}$ $\boxed{1}$ *Press* $\boxed{GRAPH}$ *Press* $\boxed{WINDOW}$*,* *Key in the value for the windows as shown in the diagram*		
2	***Find*** $\frac{dy}{dx}$ ***when x = -2*** *Press* $\boxed{MODE}$ *…….Select :* ***FLOAT 0*** *Press* $\boxed{ENTER}$		

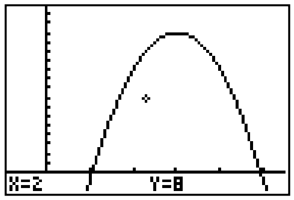
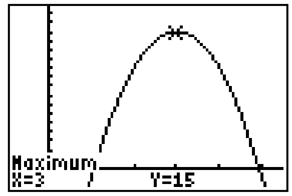
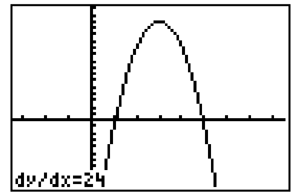
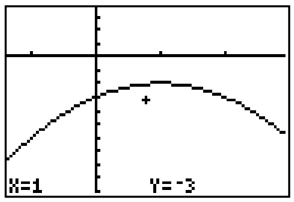
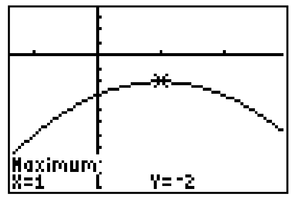
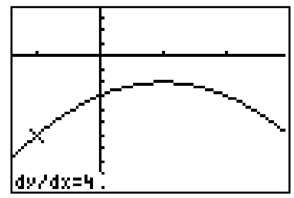
	Press [2nd] [TRACE] ***for*** [CALC] *Choose* ***6:*** $\frac{dy}{dx}$ *Press* [ENTER] *Press* [X,T,Θ,n], [CLEAR] *Key in* [(-)] [2] *Press* [ENTER]		*From the graph, as* $x=-2, y=7$ $\therefore \frac{\boldsymbol{dy}}{\boldsymbol{dx}} =$
3	***FIND MAXIMUM OR MINIMUM VALUES*** *Press* [2nd] [TRACE] ***for*** [CALC] *mode.....* *choose* ***3: minimum*** *Move the cursor to the* ***left , nearest to the minimum point*** *Press* [ENTER] *Move the cursor to the right,* ***nearest to the minimum point*** *Press* [ENTER] *Move the cursor to the* ***nearest centre*** *between left and right bound.* *Press* [ENTER]		*Since the graph is* ***U*** *shape, then, it has* ***minimum point***  $\therefore$ *Minimum point is* *(,)*

TEACHER'S NOTE

ACTIVITY 1

FILL THE TABLE WITH APPROPRIATE ANSWER

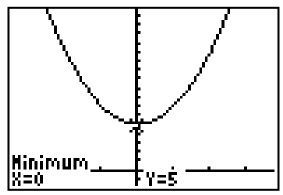
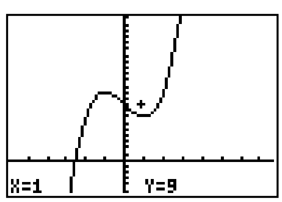
NO	FUNCTION	GRAPH	TURNING POINT/S	$\frac{dy}{dx}$ = ? WHEN
1	$y = 2x^2 + 5$ *HINT:* *Press @* 		*MINIMUM,* $x = 0, y = 5$ 	*x = -2* $\frac{dy}{dx} = -8$ 
2	$y = 4x - x^2$ *HINT:* 		*MAXIMUM,* $x = 2, y = 4$ 	*x = 6* $\frac{dy}{dx} = -8$ 
3	$y = x^3 - 3x + 9$		*MAXIMUM,* $x = -1, y = 11$ *MINIMUM,* $x = 1, y = 7$ 	*x = 4* $\frac{dy}{dx} = 45$ 

4	$f(x) = -4(x-3)^2 + 15$		MAXIMUM, $x = 3, y = 15$ 	*x = 0* $\frac{dy}{dx} = 24$ 
5	$f(x) = -x^2 + 2x - 3$		MAXIMUM, $x = 1, y = -2$ 	*x = -1* $\frac{dy}{dx} = 4$ 

STUDENT'S WORKSHEET

ACTIVITY 1

FILL THE TABLE WITH APPROPRIATE ANSWER

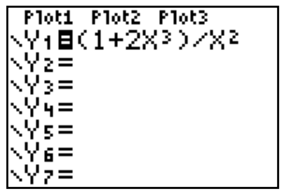
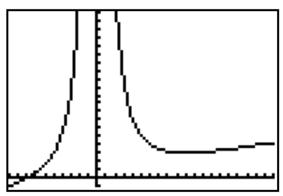
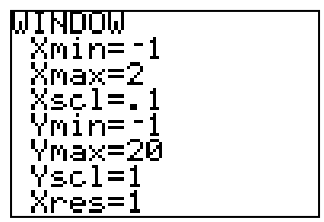
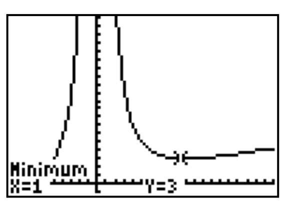
NO	*FUNCTION*	*GRAPH*	*TURNING POINT/S*	$\frac{dy}{dx}$ *= ? WHEN*
1	$y = 2x^2 + 5$		*MINIMUM,* $x = 0, y = 5$  	*x = -2* $\frac{dy}{dx} =$
2	$y = 4x - x^2$			*x = 6* $\frac{dy}{dx} =$
3	$y = x^3 - 3x + 9$	 		*x = 4* $\frac{dy}{dx} =$

4	$f(x) = -4(x-3)^2 + 15$			*x = 0* $\frac{dy}{dx} = \boxed{}$
5	$f(x) = -x^2 + 2x - 3$			*x = -1* $\frac{dy}{dx} = \boxed{}$

TEACHER'S NOTE

ACTIVITY 2

Prove that the curve $y = \dfrac{1+2x^3}{x^2}$ ***has only one turning point. Determine whether the turning point is maximum or minimum.*** *[answer: (1,3), minimum point]*

Procedure	***Screen***	***Notes***
Hints: *Press [Y=], key in the equation* $y = \dfrac{1+2x^3}{x^2}$,		
Press [GRAPH].		*(Suggested window to get the appropriate graph)* 
Press [2nd] [CALC], choose ***3:minimum,*** *Move the cursor to the left bound, [ENTER].* *Move the cursor to the right bound, [ENTER].* *Move the cursor to the minimum point,* *[ENTER].*		*[answer: (1,3), minimum point]*

STUDENT'S WORKSHEET

ACTIVITY 2

Prove that the curve $y = \dfrac{1+2x^3}{x^2}$ ***has only one turning point. Determine whether the turning point is maximum or minimum.***

Procedure	Screenshot/Keystroke	Notes
Hints: *Press* [Y=], *key in the equation* $y = \dfrac{1+2x^3}{x^2}$,		
Press [GRAPH].		
Press [2nd] [CALC], *choose* ***3:minimum,*** *move the cursor to the left bound,* [ENTER]. *Move the cursor to the right bound,* [ENTER]. *Move the cursor to the minimum point ,* [ENTER].		

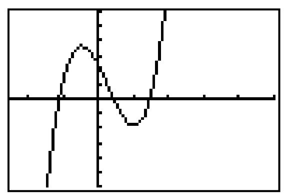
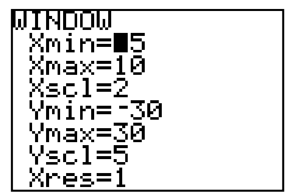
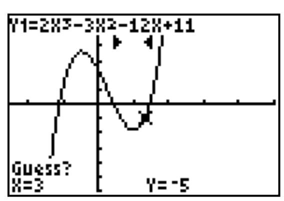
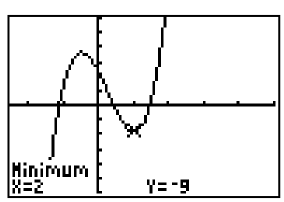
TEACHER'S NOTE

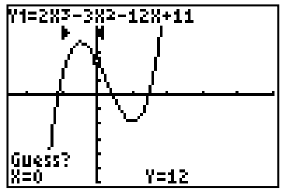
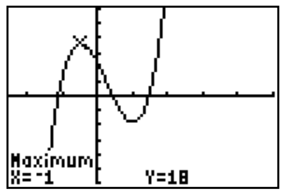
SPM Question (2005)

The equation of a curve is $y = 2x^3 - 3x^2 - 12x + 11$.

(a) Find the coordinate of the turning point of the curve. Determine whether each of the turning points is a maximum point or a minimum point.

[Answer: (-1, 18) Maximum point, (2,-9) Minimum point]

Step	Procedure	Screenshot / key-stroke	Notes
1	***DRAW THE GRAPH*** $y = 2x^3 - 3x^2 - 12x + 11$. *Press* [Y=] [2][X,T,Θ,n][^][3][−][3][X,T,Θ,n][x^2][−][1][2][X,T,Θ,n][+][1][1] *Press* [GRAPH] *Press* [WINDOW], *Key in the value for the windows as shown in the diagram*		*Suggested window:* 
2	***FIND MAXIMUM OR MINIMUM VALUES*** *Press* [2nd] [TRACE] ***for*** [CALC] *mode…..select* ***3: minimum*** *Move the cursor to the* ***left ,*** ***nearest to the minimum point*** *Press* [ENTER] *Move the cursor to the right,* ***nearest to the minimum point*** *Press* [ENTER] *Move the cursor the* ***nearest centre*** *between left and right bound.* *Press* [ENTER]		∴ *Minimum point is* ***(2,-9)***

3	***FIND MAXIMUM POINT*** *Press* [2nd] [TRACE] ***for*** [CALC] *mode.* *select* ***4: maximum*** *Move the cursor to the* ***left , nearest to the maximum point*** *Press* [ENTER] *Move the cursor to the right,* ***nearest to the maximum point*** *Press* [ENTER] *Move the cursor the* ***nearest centre*** *between left and right bound.* *Press* [ENTER]	 	*∴ Maximum point is* ***(-1,18)***

STUDENT'S WORKSHEET

SPM Question (2005)

The equation of a curve is $y = 2x^3 - 3x^2 - 12x + 11$.

(a) Find the coordinate of the turning point of the curve. Determine whether each of the turning points is a maximum point or a minimum point.

SOLUTION:

TOPIC : ***PROGRESSIONS***

SUB TOPIC : *ARITHMETIC PROGRESSIONS*

LESSON OBJECTIVE :

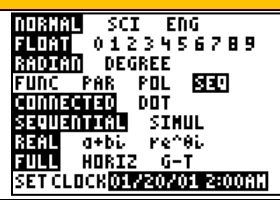
Students will be able to…

i. Determine by using formula:

a) specific terms in Arithmetic Progressions
b) the number of terms in Arithmetic Progressions
c) find the sum of the first n terms of Arithmetic Progressions

EXAMPLE QUESTION : *Given the Arithmetic sequence 1, 4, 7, 10 ... Find:*

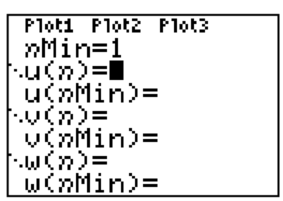
a) T_{12}
b) n *when* $T_n = 64$
c) S_{26}

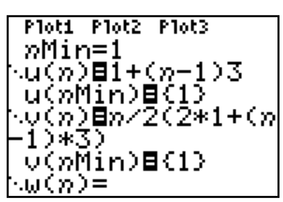
<table>
<tr><th>Step</th><th>Procedure</th><th>Screenshot / key-stroke</th><th>Notes</th></tr>
<tr><td>1</td><td>Press MODE
Move the cursor down to FUNC mode and choose SEQ function</td><td>
</td><td></td></tr>
<tr><td colspan="4">Discussion
1, 4, 7, 10 ...
Determine the value of $\boldsymbol{a}$ and $\mathbf{d}$ for the given arithmetic progression
$\boldsymbol{a} = \boxed{}$
$\boldsymbol{d} = \boldsymbol{T_n} - \boldsymbol{T_{n-1}}$
$= \boxed{} - \boxed{}$
$= \boxed{}$</td></tr>
<tr><td>2</td><td>Press Y=

$nMin = 1$

Press X,T,Θ,n to insert n symbols.

$\mu(n) = 1 + (n-1)3$
$\mu(nMin) = \{1\}$ → first term, a

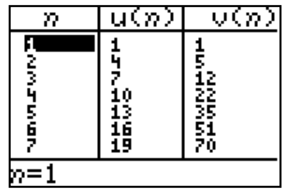
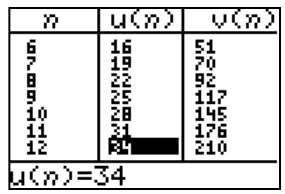
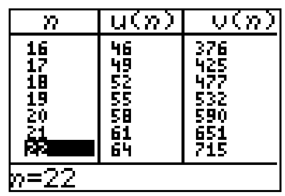
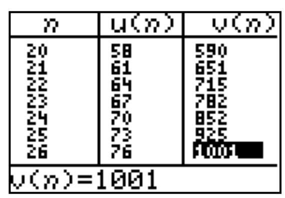
$v(n) = n/2(2*1+(n-1)*3)$
$v(nMin) = \{1\}$ → first term, a</td><td>


</td><td>$\boldsymbol{\mu(nMin) = \{1\}}$
$\boldsymbol{\mu(n) = 1 + (n-1)3}$

This is the first function that can be made, from $\boldsymbol{T_n = a + (n-1)d}$

And the second function is:
$\boldsymbol{S_n = \frac{n}{2}[2a + (n-1)d]}$

$\boldsymbol{v(n) = n/2(2*1+(n-1)*3}$
$\boldsymbol{v(nMin) = \{1\}}$</td></tr>
</table>

3	***VIEW THE SEQUENCE*** *Press* [2nd][WINDOW] *for [TBLSET] mode set the table:* ***TblStart = 1*** ***ΔTbl = 1***		
4	*Press* [2nd][GRAPH] *for [TABLE] mode*		***Remember;*** $\boldsymbol{u(n) = T_n}$ $\boldsymbol{v(n) = S_n}$
5	***TO DETERMINE THE ANSWER*** *Move the cursor down to the column* n *until* $n = 12$ $\boldsymbol{T_{12}} =$ *Move the cursor down to the column* $\mu(n)$ *until* $\boldsymbol{\mu(n) = 64}$ $\boldsymbol{n} =$ *Move the cursor down to the column* n *until* $n = 26$ $\boldsymbol{S_{26}} =$	  	$\boldsymbol{u(n) = T_n}$ $\boldsymbol{v(n) = S_n}$

TEACHER'S NOTE

Given the arithmetic sequence, Find:-

No	**Arithmetic Sequence**	T_n	n	S_n
1	2, 6, 10, 14…	$T_{20} = 78$	When $T_n = 38$, $n = 10$	$S_{16} = 512$
2	21, 18, 15, 12…	$T_{15} = -21$	When $T_n = -30$, $n = 18$	$S_{20} = -150$
3	-6, 1, 8, 15…	$T_{10} = 57$	When $T_n = 85$, $n = 14$	$S_{20} = 1210$
4	-12, -9, -6, -3…	$T_{25} = 60$	When $T_n = 45$, $n = 20$	$S_{10} = 15$
5	$\frac{1}{3}, \frac{7}{12}, \frac{5}{6}, \frac{13}{12} \ldots$	$T_8 = 2.0833$	When $T_n = 4.3333$, $n = 17$	$S_{15} = 8.5$

STUDENT'S WORKSHEET

Given the arithmetic sequence, Find:-

No	Arithmetic Sequence	T_n	n	S_n
1	2, 6, 10, 14…	$T_{20} =$	When $T_n = 38$, $n =$	$S_{16} =$
2	21, 18, 15, 12…	$T_{15} =$	When $T_n = -30$, $n =$	$S_{20} =$
3	-6, 1, 8, 15…	$T_{10} =$	When $T_n = 85$, $n =$	$S_{20} =$
4	-12, -9, -6, -3…	$T_{25} =$	When $T_n = 45$, $n =$	$S_{10} =$
5	$\frac{1}{3}, \frac{7}{12}, \frac{5}{6}, \frac{13}{12}\ldots$	$T_8 =$	When $T_n = 4.3333$, $n =$	$S_{15} =$

TEACHER'S NOTE

ENRICHMENTS
SPM 2004 (PAPER 1: NO. 11)

The volume of water in a tank is 450 liters on the first day. Subsequently, 10 liters of water is added to the tank every day.

Calculate the volume, in liters, of water in the tank at the end of the 7^{th} day.

[2 marks]

Answer: $T_7 = 510$

SPM 2005 (PAPER 1: NO.11)

The first three terms of an arithmetic progression are 5, 9, 13. Find

a) *the common difference of the progression*
b) *the sum of the first 20 terms after the 3^{rd} term*

[4 marks]

Answer:
a) $d = 4$
b) 1100

SPM 2005 (PAPER 2: SECTION A: NO.3)

Diagram 1 shows part of an arrangement of bricks of equal size.

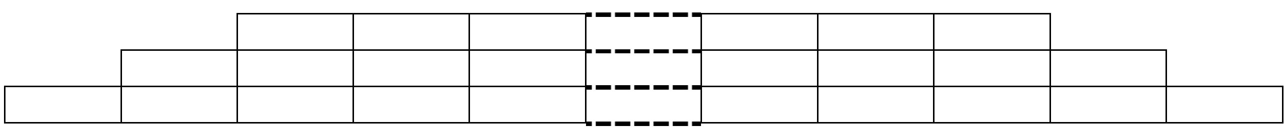


Diagram 1

The number of bricks in the lowest row is 100. For each of the rows, the number of bricks is 2 less than in the row below. The height of each bricks is 6 cm.

Ali builds a wall by arranging bricks in this way. The number of bricks in the highest row is 4. Calculate

a) *the height, in cm, of the wall* *[3 marks]*
b) *the total price of the bricks used if the price of one brick is 40 sen* *[3 marks]*

Answer:
a) *294 cm*
b) *RM 1019.20*

STUDENT'S WORKSHEET

ENRICHMENTS

SPM 2004 (PAPER 1: NO. 11)

The volume of water in a tank is 450 liters on the first day. Subsequently, 10 liters of water is added to the tank every day.

Calculate the volume, in liters, of water in the tank at the end of the 7^{th} day.

[2 marks]

SPM 2005 (PAPER 1: NO.11)

The first three terms of an arithmetic progression are 5, 9, 13. Find

a) *the common difference of the progression*
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[4 marks]

SPM 2005 (PAPER 2: SECTION A: NO.3)

Diagram 1 shows part of an arrangement of bricks of equal size.

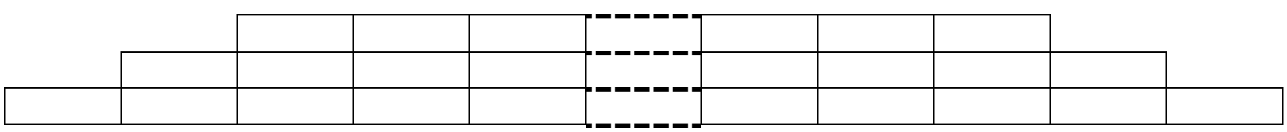


Diagram 1

The number of bricks in the lowest row is 100. For each of the rows, the number of bricks is 2 less than in the row below. The height of each bricks is 6 cm.

Ali builds a wall by arranging bricks in this way. The number of bricks in the highest row is 4. Calculate

a) *the height, in cm, of the wall* *[3 marks]*
b) *the total price of the bricks used if the price of one brick is 40 sen* *[3 marks]*

TOPIC : ***PROGRESSIONS***

SUB TOPIC : *GEOMETRIC PROGRESSIONS*

LESSON OBJECTIVE :

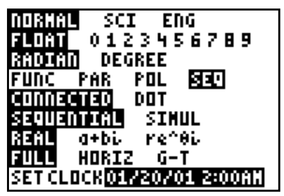
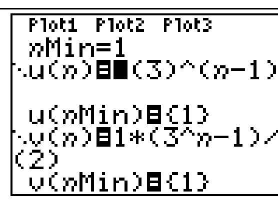
Students will be able to…

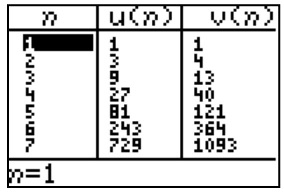
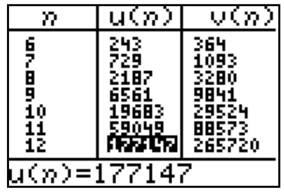
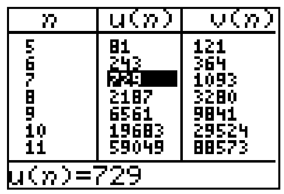
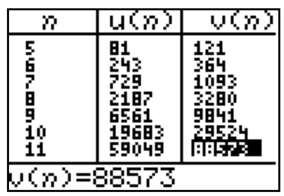
i. Determine by using formula:

- *a) specific terms in Geometric Progressions*
- *b) the number of terms in Geometric Progressions*
- *c) find the sum of the first n terms of Geometric Progressions*

EXAMPLE : *Given the Geometric sequence 1, 3, 9, 27 … Find*

- *a)* T_{12}
- *b)* n *when* T_n *= 729*
- *c)* S_{11}

Step	***Procedure***	***Screenshot / key-stroke***	***Notes***
1	*Press* $\boxed{\text{MODE}}$ *Choose **SEQ** function*		
Discussion *Determine the value of* $\boldsymbol{a}$ *and* $\boldsymbol{r}$ *for the given geometric progression* $\boldsymbol{a} = \boxed{}$ $\boldsymbol{r} = \dfrac{\boldsymbol{T_n}}{\boldsymbol{T_{n-1}}} = \dfrac{\boxed{}}{\boxed{}}$ $= \boxed{}$			
2	*Press* $\boxed{\text{Y=}}$ $nMin = 1$ *Press* $\boxed{X,T,\Theta,n}$ *to insert n symbols.* $\mu(n) = 1(3)^{n-1}$ $\mu(nMin) = \{1\}$ → *first term, a*		$\boldsymbol{\mu(n) = 1(3)^{n-1}}$ $\boldsymbol{\mu(nMin) = \{1\}}$ *This is the first function that can be made, from* $T_n = ar^{(n-1)}$ *And the second function is:* $S_n = \dfrac{a(r^n - 1)}{r - 1}$

	$v(n) = 1 * (3^{n-1})$/(3-1) $v(nMin) = \{1\}$ → *first term, a*		$\boldsymbol{v(n) = 1 * (3^{n-1})}$***/(3-1)*** $\boldsymbol{v(nMin)} = \{\mathbf{1}\}$
3	***<u>TO DETERMINE THE ANSWER</u>*** *Move the cursor down to the column* n *until* $\boldsymbol{n = 12}$ $\boldsymbol{T_{12}} =$ *Move the cursor down to the column* $\mu(n)$ *until* $\boldsymbol{\mu(n) = 729}$ $\boldsymbol{n} =$ *Move the cursor down to the column* $v(n)$ *until* $\boldsymbol{n = 11}$ $\boldsymbol{S_{11}} =$	  	*STEPS FOR GETTING THE ANSWER IS SIMILAR WITH ARITHMETIC PROGRESSION.* ***BE CAREFUL WHEN KEY IN <u>THE FORMULA</u> FOR ARITHMETIC OR GEOMETRIC PROGRESSION***

TEACHER'S NOTE

Given the Geometric sequence, Find:-

No	Geometric Sequence	T_n	n	S_n
1	192, -96, 48, -24, …	$T_7 = 3$	When $T_n = 0.1875$, $n = 11$	$S_7 = 129$
2	2, 6, 18, 54, …	$T_{10} = 39366$	When $T_n = 1458$, $n = 7$	$S_8 = 6560$
3	20, 10, 5 …	$T_9 = 0.07813$	When $T_n = \frac{5}{8}$, $n = 6$	$S_{17} = 40$
4	0.1,-0.3,0.9 …	$T_{14} = -1968$	When $T_n = -218.7$, $n = 8$	$S_{20} = -53144$
5	1458, 486, 162, 54, …	$T_{10} = 0.7407$	When $T_n = 2$, $n = 7$	$S_{10} = 2187$

ENRICHMENTS

1. *In the progression 5, 10, 20, 40,… Find the least number of terms required such that their sum exceeds 1000.*

Answer: 8

SPM 2005 (PAPER 1: NO.12)

The sum of the first n terms of the geometric progression 8, 24, 72,… is 8744. Find

a) *the common ratio of the progression*
b) *the value of n* *[4 marks]*

Answer: a) r = 3
b) n = 7

STUDENT'S WORKSHEET

Given the Geometric sequence, Find:-

No	Geometric Sequence	T_n	n	S_n
1	192, -96, 48, -24, …	$T_7 =$	When $T_n = 0.1875$, $n =$	$S_7 =$
2	2, 6, 18, 54, …	$T_{10} =$	When $T_n = 1458$, $n =$	$S_8 =$
3	20, 10, 5 …	$T_9 =$	When $T_n = \frac{5}{8}$, $n =$	$S_{17} =$
4	0.1,-0.3,0.9 …	$T_{14} =$	When $T_n = -218.7$, $n =$	$S_{20} =$
5	1458, 486, 162, 54, …	$T_{10} =$	When $T_n = 2$, $n =$	$S_{10} =$

ENRICHMENTS

1. *In the progression 5, 10, 20, 40,… Find the least number of terms required such that their sum exceeds 1000.*

SPM 2005 (PAPER 1: NO.12)

The sum of the first n terms of the geometric progression 8, 24, 72,… is 8744.
Find

a) the common ratio of the progression
b) the value of n *[4 marks]*

TOPIC **:** ***LINEAR LAW***

SUB TOPIC : *LINE OF BEST FIT*

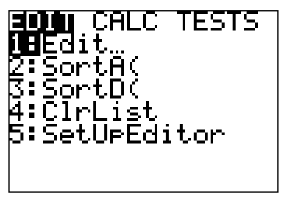
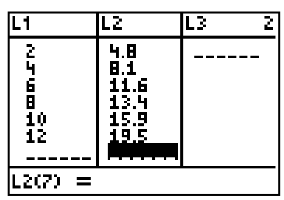
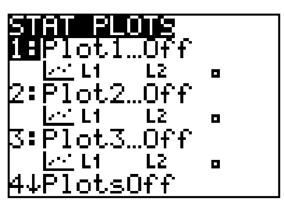
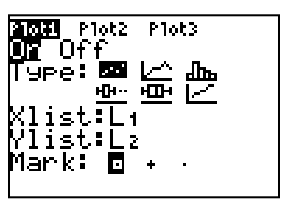
LESSON OBJECTIVES **:**

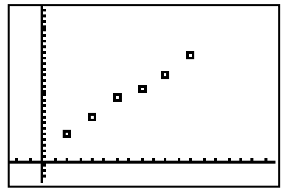
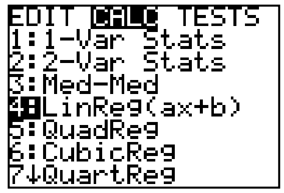
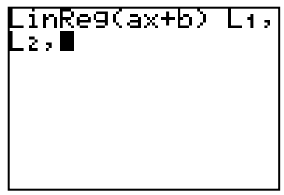
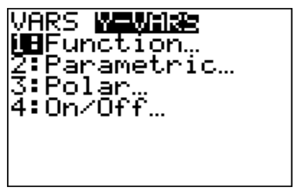
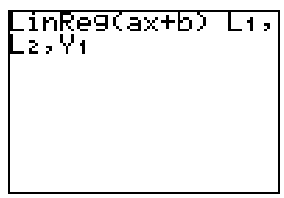
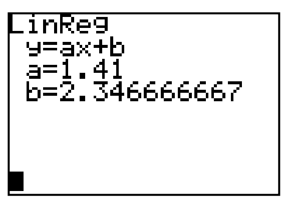
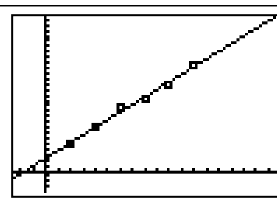
Students will be able to…

- i. *Draw line of best fit by inspection of given data.*
- ii. *Write equation for lines of best fit.*
- iii. *Determine values of variables from:*
  - a) *lines of best fit*
  - b) *equations of lines of best fit*

EXAMPLE **:** *Draw a line of best fit from the given set of data.*

x	*2*	*4*	*6*	*8*	*10*	*12*
y	*4.8*	*8.1*	*11.6*	*13.4*	*15.9*	*19.5*

Step	***Procedure***	***Screenshot / key-stroke***	***Notes***
1	***KEY-IN VALUES X AND Y*** *Press [STAT]* *Choose **1.EDIT…*** *[ENTER]*	 	$L_1 = x, L_2 = y$
2	***PLOTTING THE GRAPH*** *Press [2nd][Y=] for [STAT PLOT] mode* *Choose **1** : **Plot 1*** *[ENTER]* *Choose the P**lotter Graph*** *[ENTER]*	 	

3	*Press* [ZOOM] ***Choose 9 : ZOOM STAT***		
4	***TO OBTAIN THE EQUATION OF THE LINEAR FUNCTION*** *Press* [STAT] *Choose*[CALC]. *Choose* ***4: LinReg (ax+b)****,* ***Press*** [2nd] [L1] [,] [2nd] [L2] [,] *Press* [VARS] *Choose* [Y-VARS] *Choose* ***1: Function*** *Choose* ***1: Y1*** [ENTER] *Press* [ENTER] *again.*	    	
5	***DRAW LINE OF BEST FIT*** *Press* [GRAPH]		

TEACHER'S NOTE

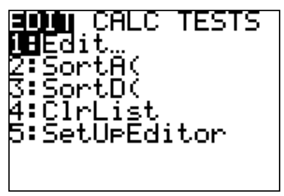
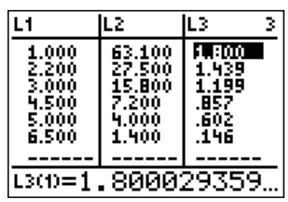
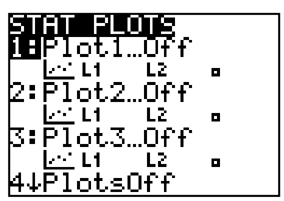
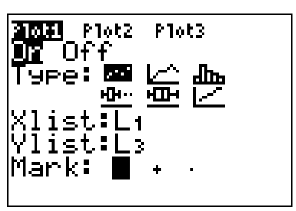
1 The table shows the experimental values of two variables x and y.

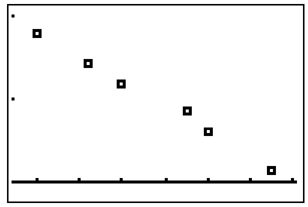
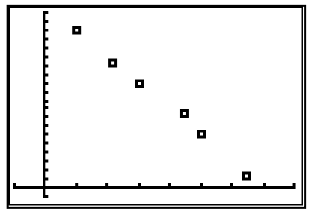
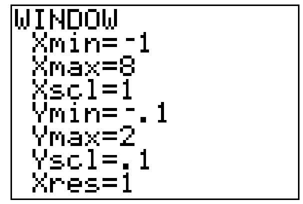
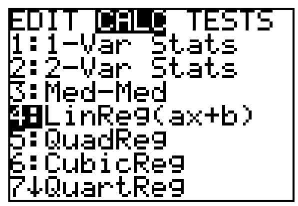
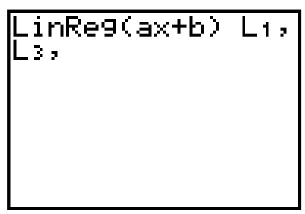
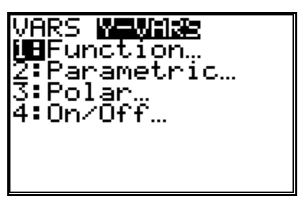
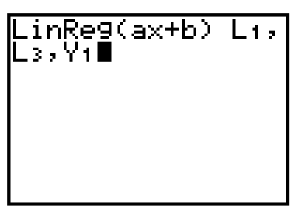
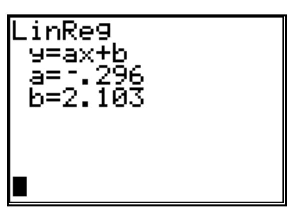
x	1.0	2.2	3.0	4.5	5.0	6.5
y	63.1	27.5	15.8	7.2	4.0	1.4

It is known that y and x is related by an equation $y = pq^{-x}$ where p and q are constants.

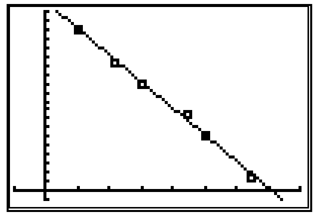
a) **Plot** a graph of **$\log_{10} y$** against **x** and **draw a line of best fit**.
b) Use your graph to find **the value of**
   i. **p**
   ii. **q**

ANSWER:

Step	Procedure	Screenshot / key-stroke	Notes
1	**KEY IN THE VALUES** Press [STAT] Choose **1.EDIT…** [ENTER] Move the cursor to **L3** Press [LOG] [2nd] [L2]	 	$L_1 = x,$ $L_2 = y,$ $L_3 = \log y$ Press [MODE]. Move the cursor to FLOAT Choose 3 for three decimal places. [ENTER]
2	**PLOTTING THE GRAPH** Press [2nd][Y=] for [STAT PLOT] mode Choose 1: **Plot 1** [ENTER] Choose the P**lotter Graph** Move the cursor to **Ylist** Press [2nd] [L3] [ENTER]	 	Plot $\log_{10} y$ against x

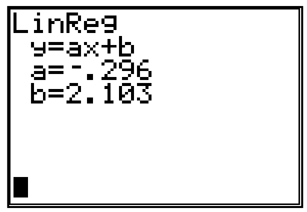
3	*Press* [ZOOM] ***Choose 9 : ZOOM STAT*** ***OR*** ***Press WINDOW***	 	*Suggested Window* 
4	***<u>TO OBTAIN THE EQUATION OF LINEAR FUNCTION</u>*** *Press* [STAT] *Choose*[CALC]. *Choose* ***4: LinReg (ax+b)***, *Press* [2nd] [L1] [,] [2nd] [L3] [,] *Press* [VARS] *Choose* [Y-VARS] *Choose* ***1: Function*** [ENTER] *Choose* ***1: Y1*** [ENTER] *Press* [ENTER]	    	

<table>
<tr>
<td>5</td>
<td><u>DRAW LINE OF BEST FIT</u>

Press GRAPH</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td><u>FINALIZE THE ANSWER</u>

From the equation,

$\log_{10} p$ = y-intercept
= 2.103
<u>p = 126.77</u>

$-\log_{10} q$ = Gradient
= -0.296
<u>q = 1.977</u></td>
<td></td>
<td>Change $y = pq^{-x}$ into linear equation in form $Y = mX + c$

$\log_{10} y = (-\log q)\, x + \log_{10} p$

a = gradient = m
b = Y-intercept = c</td>
</tr>
</table>

STUDENT'S WORKSHEET

1 *The table shows the experimental values of two variables x and y.*

x	1.0	2.2	3.0	4.5	5.0	6.5
y	63.1	27.5	15.8	7.2	4.0	1.4

It is known that y and x is related by an equation $y = pq^{-x}$ where p and q are constants.

a) ***Plot** a graph of **$\log_{10} y$** against **x** and **draw a line of best fit**.*
b) *Use your graph to find **the value of***

i. ***p,***
ii. ***q.***

ANSWER :

Step	Procedure	Notes
1	*Key in values*	*Construct a new table* <table><tr><td>x</td><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td>y</td><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td>Log y</td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table>
2	*Plotting graph*	*Use graph paper*
3	*Draw line of best fit* *- Line passes through as many*	*Line of best fit – line passes through as many point as possible.*
4	*Find the value of p and q*	*Change the given equation into linear equation in the form of Y = mX + c.*

TEACHER'S NOTE

2. *Table shows the values of two variables, x and y, obtained from an experiment. The variables x and y are related by the equation $y = kx + \frac{h}{kx}$, where k and h are constants.*

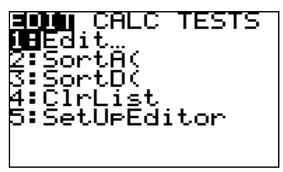
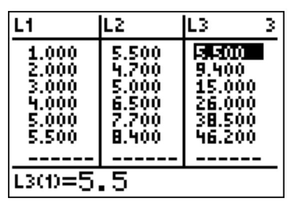
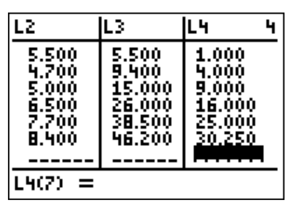
x	1.0	2.0	3.0	4.0	5.0	5.5
y	5.5	4.7	5.0	6.5	7.7	8.4

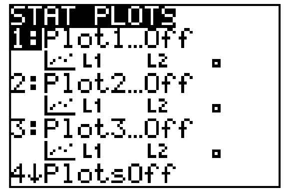
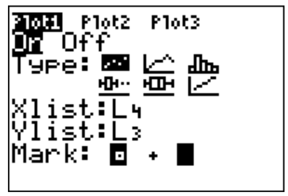
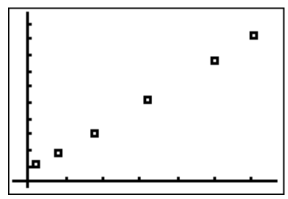
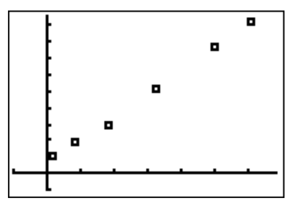
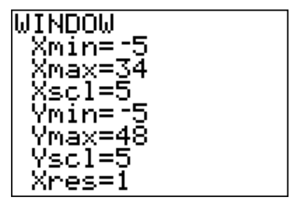
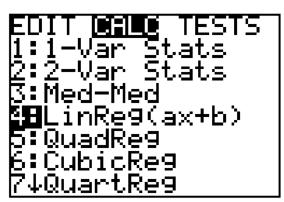
(a) *Plot a graph of yx against x^2, by using a scale of 2 cm to 5 units on both axes. Hence,* ***draw a line of best fit.***

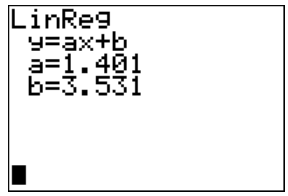
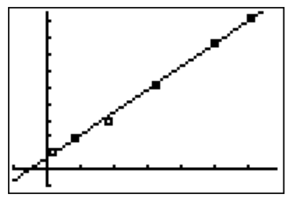
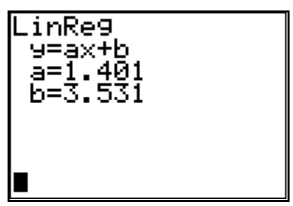
(b) *Use your graph to from (a) to find the value of*

(i) ***k,***
(ii) ***h.***

ANSWER:

Step	***Procedure***	***Screenshot / key-stroke***	***Notes***
1	***KEY IN THE VALUES*** *Press [STAT]* *Choose **1.EDIT…*** *[ENTER]* *Key in the value of x in L_1* *Key in the value of y in L_3* *Move the cursor to L_3* *Press [2nd] [1] **(for** [L1]**)** [×] [2nd][2] (for [L2])* *[ENTER]* *Move the cursor to [L4]* *Press [2nd] [1] (for [L1]) [x^2]* *[ENTER]*	  	$L_1 = x,$ $L_2 = y,$ $L_3 = xy$ $L_4 = x^2$ *Press [MODE]. Move the cursor to **FLOAT** Choose 3 for three decimal places.* *[ENTER]*

2	***PLOTTING THE GRAPH*** *Press [2nd][Y=] for [STAT PLOT] mode* *Choose **1:Plot 1*** *[ENTER]* *Choose the P**lotter Graph*** *Move the cursor to Xlist* *Press [2nd] [L4]* *Move the cursor to Ylist* *Press [2nd] [L4] [L3]* *[ENTER]*	 	*Plot xy against x^2*
3	*Press [ZOOM]* *Choose **9 : ZOOM STAT*** ***OR*** *Press [WINDOW]*	 	*Suggested Window* 
4	***TO OBTAIN THE EQUATION OF LINEAR FUNCTION*** *Press [STAT]* *Choose[CALC].* *Choose **4: LinReg (ax+b)**,*		

	Press [2nd] [4] ***(for*** [L4]***)*** [,] [2nd] [3] ***(for*** [L3] [,] *Press* [VARS] *Choose [Y-VARS]* *Choose* ***1: Function*** [ENTER] *Choose* ***1: Y1*** [ENTER] *Press* [ENTER]	 	
5	***DRAW LINE OF BEST FIT*** *Press* [GRAPH]		
6	***FINALIZE THE ANSWER*** *From the equation* *k = gradient* ***k = 1.401*** $\frac{h}{k} = y - \text{intercept}$ $= \underline{\mathbf{3.531}}$ $\therefore h = 3.531 \times k$ $\underline{h = \mathbf{4.95}}$	***Can you define which is Y, m, X, and c?*** 	*Change the equation* $y = kx + \frac{h}{kx}$ *into linear equation in form of* ***Y = m X + c*** $xy = kx^2 + \frac{h}{k}$ *a = gradient = m* *b = Y-intercept = c*

STUDENT'S WORKSHEET

2. Table shows the values of two variables, x and y, obtained from an experiment. The variables x and y are related by the equation $y = kx + \frac{h}{kx}$, where k and h are constants.

x	1.0	2.0	3.0	4.0	5.0	5.5
y	5.5	4.7	5.0	6.5	7.7	8.4

(b) Plot a graph of yx against x^2, by using a scale of 2 cm to 5 units on both axes. Hence, **draw a line of best fit.**

(c) Use your graph to from (a) to find the value of

(i) **k,**
(ii) **h.**

ANSWER:

Step	Procedure	Notes
1	Key in values	Construct a new table <table><tr><td>x</td><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td>y</td><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td>xy</td><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td>x^2</td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table>
2	Plotting graph	Use graph paper
3	Draw line of best fit - Line passes through as many	Line of best fit – line passes through as many point as possible.
4	Find the value of p and q	Change the given equation into linear equation in the form of **Y = mX + c.** ***Can you define which is Y, m, X, and c?***

STUDENT'S WORKSHEET

ENRICHMENTS

1. *Table shows the values of two variables, x and y, obtained from an experiment.*

   Variables and y are related by the equation $y = 2kx^2 + \frac{p}{k}x$*, where p and k are constants.*

x	2	3	4	6	8	7
y	8	13.2	20	27.5	36.6	45.5

   a) *Plot* $\frac{y}{x}$ *against x, using a scale of 2 cm to 1 unit on both axes. Hence, draw the line of best fit.*

   b) *Use your graph in (a), to find the value of*

      (i) p,
      (ii) k,
      (iii) y *when* y *= 1.2*

2. *Table shows the values of two variables, x and y, obtained from an experiment.*
   Variables x and y are related by the equation $y = hk^{2x}$*, where h and k are constants.*

x	1.5	3.0	4.5	6.0	7.5	9.0
y	2.51	3.24	4.37	5.75	7.76	10.00

   a) *Based on Table, construct a table for the values of $\log_{10} y$.*

   b) *Plot $\log_{10} y$ against x, using a scale of 2 cm to 1 unit on the x -axis and 2 cm to 0.1 unit on the $\log_{10} y$ axis. Hence, draw the line of best fit.*

   c) *Use the graph in (b) to find the value of:*

      (i) x *when* $y = 4.8$,
      (ii) h,
      (iii) k.

TOPIC : ***INTEGRATION***

SUBTOPIC : DEFINE INTEGRALS

LESSON OBJECTIVE :

Students will be able to…

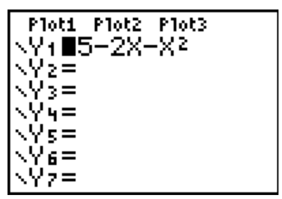
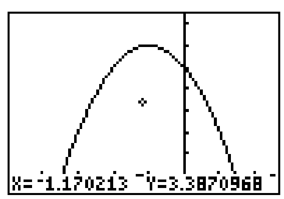
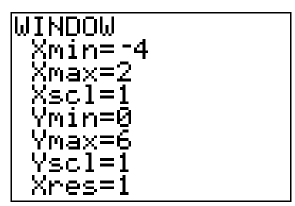
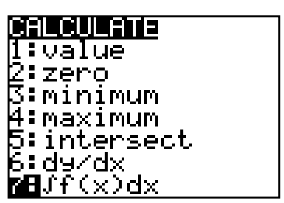
i. Understand and use the concept of definite integrals

ii. Determine the area under a curve using definite integrals

EXAMPLE QUESTION : *Compute:*

a) $\int_{-3}^{1}(5-2x-x^2)dx$

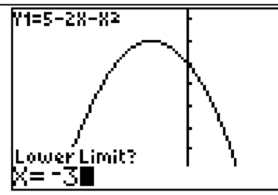
b) *Find the area bounded by the curve* ***y = 5 – 2x – x²*** *and the x-axis such that* $-3 \leq x \leq 1$

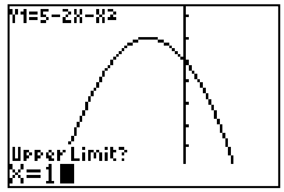
Step	***Procedure***	***Screenshot / key-stroke***	***Notes***
Questions for discussion *i. Do you think the example questions for (a) and (b) are the same? Why?* *ii. What do you understand about the word 'definite'?*			
1	***Plot the graph*** $5-2x-x^2$ *Press* [Y=] [5][−] [2][X,T,Θ,n][−] [X,T,Θ,n][x²]		
2	*Press* [GRAPH]		
3	***Find*** $\int_{-3}^{1}(5-2x-x^2)dx$ *Press* [2nd] [TRACE] *for* [CALC] *mode* *Choose **7:*** $\int f(x)dx$		

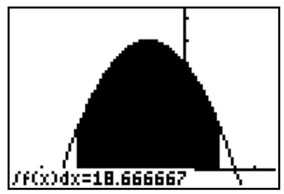
<table>
<tr>
<td></td>
<td>
Key in lower limit = -3
[ENTER]

Key in upper limit = 1
[ENTER]

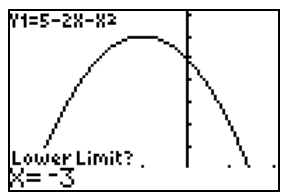
The value of the integral is

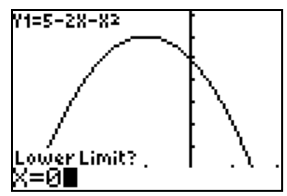
<u>Therefore, the area bounded by the curve is</u>
</td>
<td>


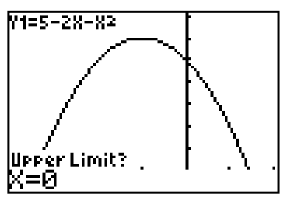


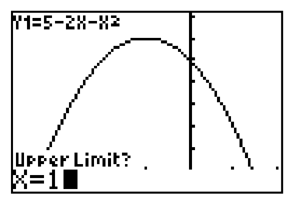


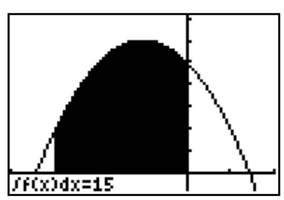
</td>
<td>
Press [2nd] [PRGM] for [DRAW] mode
Choose 1:ClrDraw
before proceed to next question.
</td>
</tr>
<tr>
<td>4</td>
<td>
Lower Limit and Upper Limit can be adjusted flexibly, based on the question asked.

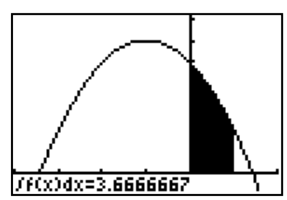
For example:
$$\boldsymbol{area} = \int_{-3}^{0} \boldsymbol{f(x)dx} + \int_{0}^{1} \boldsymbol{f(x)\,d}$$
$=$ $+$
$=$
</td>
<td colspan="2">






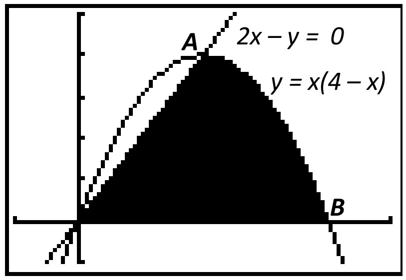






</td>
</tr>
</table>

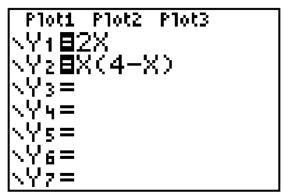
TEACHER'S NOTE

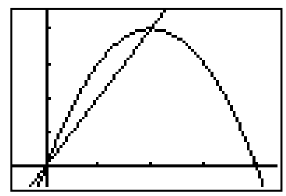


The diagram shows the area bounded by the curve $y = x(4 - x)$, the line $2x - y = 0$ and the x-axis. Find:-

a) *The coordinates of points **A** and **B**.*
b) *The **area** of shaded region.*

<table>
<tr><th>Step</th><th>Procedure</th><th>Screenshot / key-stroke</th></tr>
<tr><td>1</td><td>Key in the relevant equations

Plot the Graph</td><td>

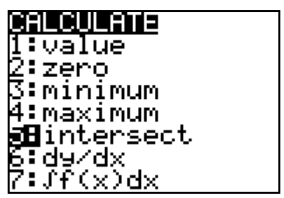
</td></tr>
<tr><td>2</td><td>To find the Intersection

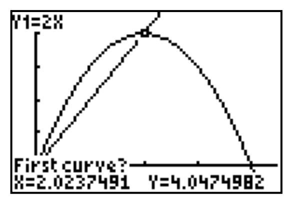
[2nd][TRACE] for [CALC] mode
Choose 5:intersect

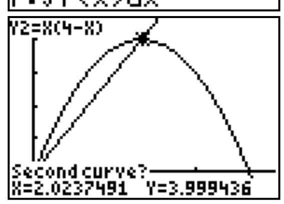
Choose first curve?
[ENTER]
(the cursor must be located near to the intersection point)

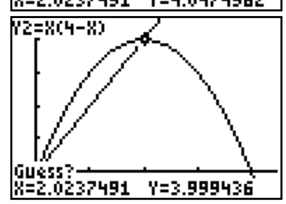
Choose second curve?
[ENTER]
(the cursor must be located near to the intersection point)

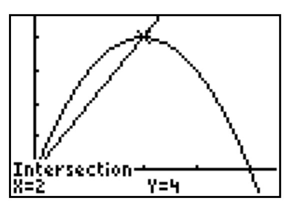
Choose Guess?
[ENTER]

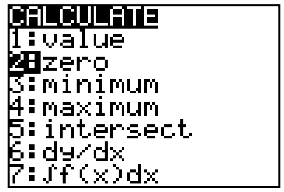
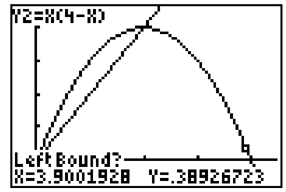
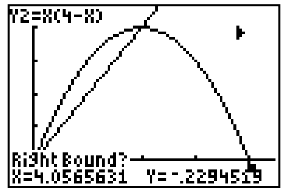
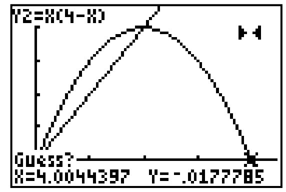
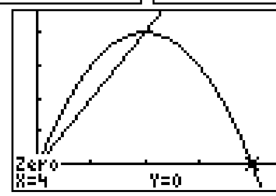
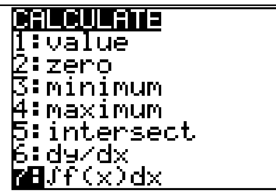
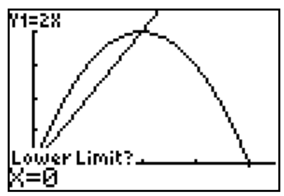
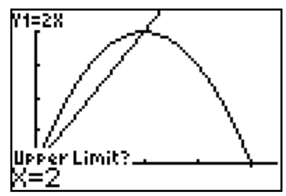
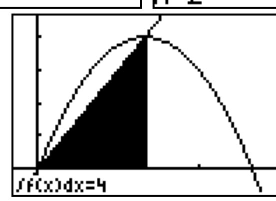
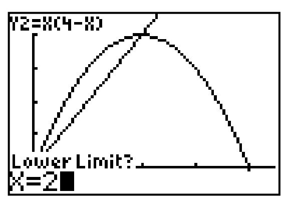
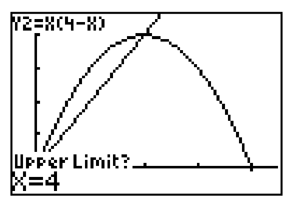
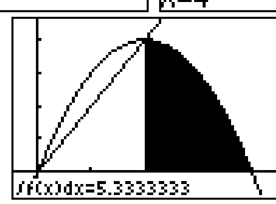
Intersection is $(2,4)$.
$\therefore A = (2,4)$</td><td>



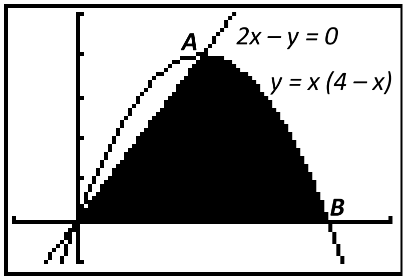





</td></tr>
</table>

3	*To find the ZERO point* *[2nd][TRACE] for [CALC] mode* *Choose **2:zero*** *Choose **Left Bound?*** ***(Bring the cursor nearby the x-axis)*** *Choose **Right Bound?*** ***(Bring the cursor nearby the x-axis)*** *Choose **Guess?*** ***(Bring the cursor between Left-Bound and Right-Bound)*** *The x-intercept is **4*** $\therefore \boldsymbol{B} = (\mathbf{4}, \mathbf{0})$	
4	*To find the area of shaded Region* *[2nd][TRACE] for [CALC] mode* *Choose **7:**$\int \boldsymbol{f}(\boldsymbol{x})\boldsymbol{dx}$* **Plot for $\int_0^2 f_1(x)dx + \int_2^4 f_2(x)\,dx$** ***(Be careful on selecting the function)*** *The **area** of shaded region* $= 4 + 5.333$ $\underline{= \mathbf{9.333}}$	

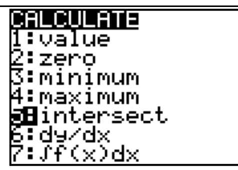
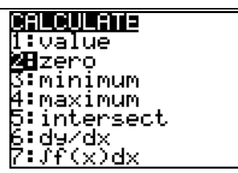
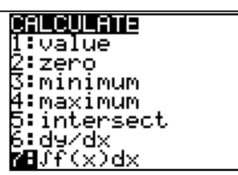
STUDENT'S WORKSHEET



The diagram shows the area bounded by the curve $y = x(4 - x)$*, the line* $2x - y = 0$ *and the x-axis. Find:-*

a) The coordinates of points ***A*** *and* ***B.***

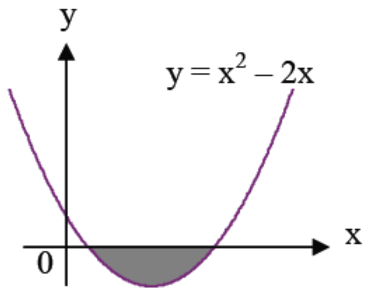
b) The ***area*** *of shaded region.*

Step	***Procedure***	***Screenshot / key-stroke***
1	*Key in the relevant equations* *Plot the Graph*	
2	*Find the Intersection* *[2nd][TRACE] for [CALC] mode* *Choose* ***5:intersect***	
3	*Find the* ***ZERO*** *point* *[2nd][TRACE] for [CALC] mode* *Choose* ***2:zero***	
4	*Find the area of shaded Region* *[2nd][TRACE] for [CALC] mode* *Choose* ***7:***$\int f(x)dx$	

TEACHER'S NOTE

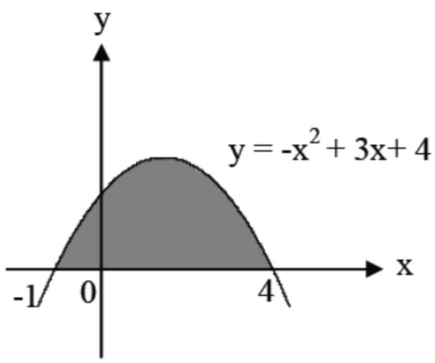
ENRICHMENT

1. Find the area of the shaded region in the diagram



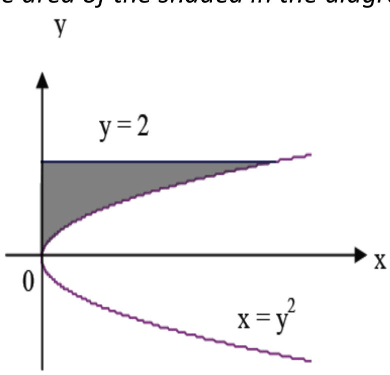
Answer: $1\frac{1}{3}unit^2$

2. Find the area of the shaded region in the diagram.



Answer: $20\frac{5}{6}unit^2$

3. Find the area of the shaded in the diagram.

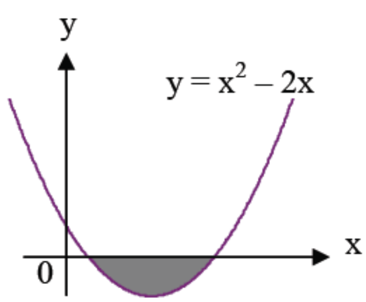


Answer: $2\frac{2}{3}unit^2$

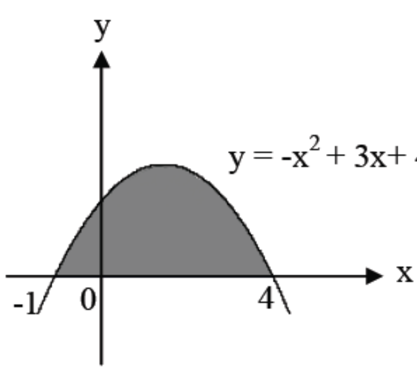
STUDENT'S WORKSHEET

ENRICHMENT

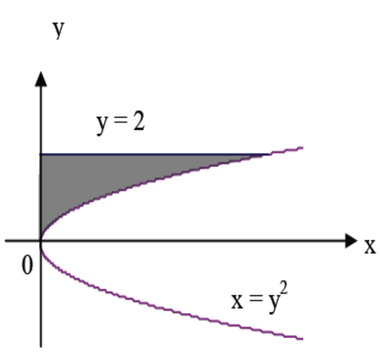
1. *Find the area of the shaded region in the diagram*



2. *Find the area of the shaded region in the diagram.*



3. *Find the area of the shaded in the diagram.*



TOPIC : ***TRIGONOMETRIC FUNCTIONS***

SUBTOPIC : GRAPHS OF SINE, COSINE AND TANGENT FUNCTIONS.

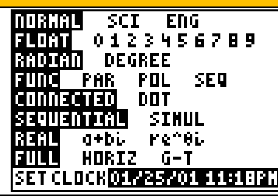
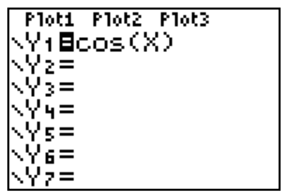
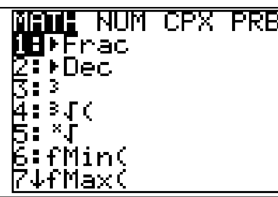
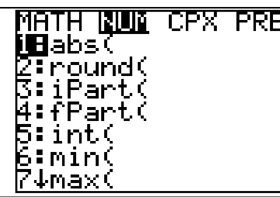
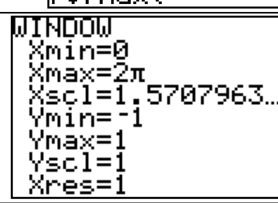
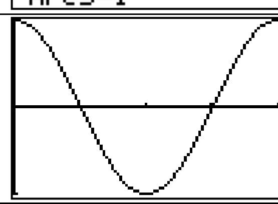
LESSON OBJECTIVE :

Students will be able to…

i. Draw and sketch graphs of trigonometric functions.

$$y = c + a \cos bx$$

EXAMPLE QUESTION : ***Draw and sketch graph of $y = \cos x$ for $0 \leq x \leq 360°$***

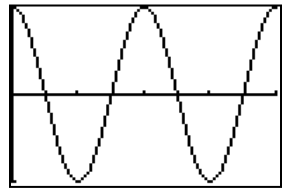
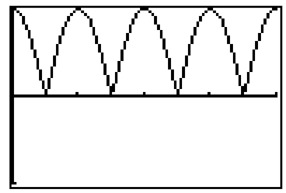
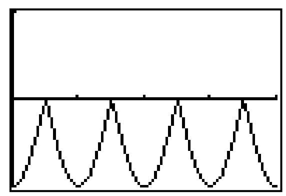
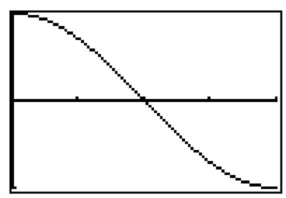
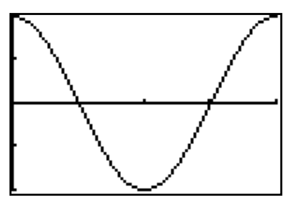
Step	Procedure	Screenshot / key-stroke	Notes
1	*Press* [MODE] *and choose* ***RADIAN***		*Make sure the RAM is clear*
2	*Press* [Y=] *Key in the trigonometry function.* $y = \cos x$ *Press* [COS] [X,T,Θ,n] [ )]		
	IF THE FUNCTION REQUIRE ***ABSOLUTE VALUE****;* *Press* [MATH] *Choose* ***NUM*** *Choose* ***1:abs(***		
3	*Press* [WINDOW] *key in WINDOW setting with appropriate value*		*Xmax = 2π = 360°,* *since* $0 \leq x \leq 360°$ *Xscl =* $\frac{\pi}{2}$ *Ymin and Ymax varies accordingly.*
4	*Press* [GRAPH] *to view the graph*		

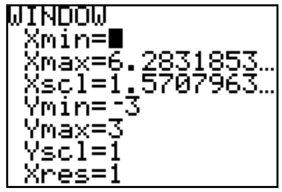
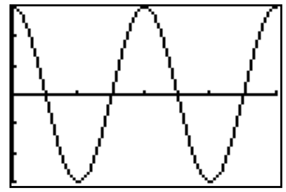
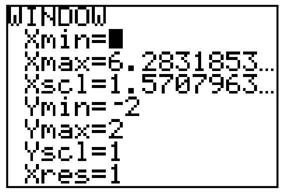
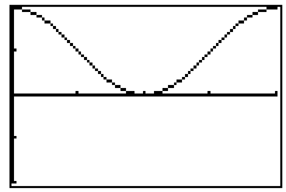
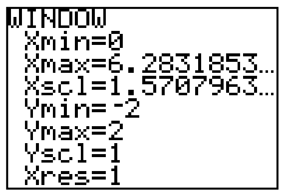
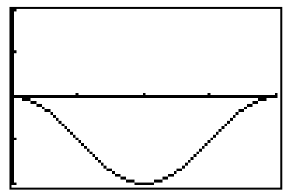
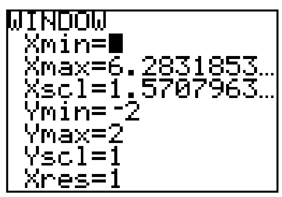
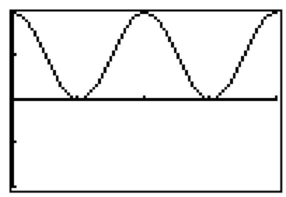
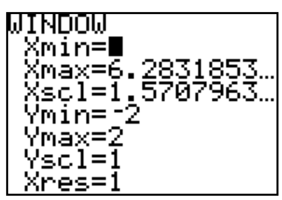
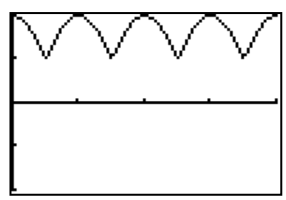
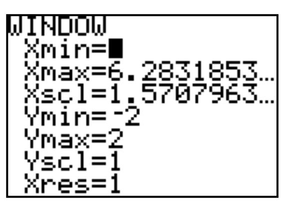
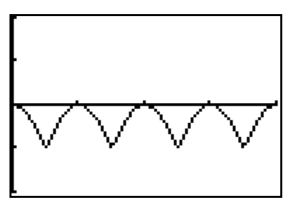
Investigation

Graph	Maximum	Minimum	Amplitude	No of Cycle
$y = \cos x$	1	-1	1	1

TEACHER'S NOTE

Draw and sketch graph of given functions for $0 \leq x \leq 360°$ by using graphing calculator

No	Function	WINDOW setting	Graph
1	$y = cos\,2x$	WINDOW Xmin=0 Xmax=6.2831853… Xscl=1.5707963… Ymin=-1 Ymax=1 Yscl=1 Xres=1	
2	$y = \|cos\,2x\|$	WINDOW Xmin=0 Xmax=6.2831853… Xscl=1.5707963… Ymin=-1 Ymax=1 Yscl=1 Xres=1	
3	$y = -\|cos\,2x\|$	WINDOW Xmin= Xmax=6.2831853… Xscl=1.5707963… Ymin=-1 Ymax=1 Yscl=1 Xres=1	
4	$y = cos\frac{1}{2}x$	WINDOW Xmin= Xmax=6.2831853… Xscl=1.5707963… Ymin=-1 Ymax=1 Yscl=1 Xres=1	
5	$y = 2\,cos\,x$	WINDOW Xmin= Xmax=6.2831853… Xscl=1.5707963… Ymin=-2 Ymax=2 Yscl=1 Xres=1	

6	$y = 3\cos 2x$		
7	$y = \cos x + 1$		
8	$y = \cos x - 1$		
9	$y = \cos 2x + 1$		
10	$y = \lvert\cos 2x\rvert + 1$		
11	$y = \lvert\cos 2x\rvert - 1$		

TEACHER'S NOTE

INVESTIGATION

No	Graph	Maximum	Minimum	Amplitude	No of Cycle
1	$y = cos\,2x$	1	-1	1	2
2	$y = \|cos\,2x\|$	1	0	1	2
3	$y = -\|cos\,2x\|$	0	-1	1	2
4	$y = cos\,\frac{1}{2}x$	1	-1	1	$\frac{1}{2}$
5	$y = 2\,cos\,x$	2	-2	2	1
6	$y = 3\,cos\,2x$	3	-3	3	2
7	$y = cos\,x + 1$	2	0	1	1
8	$y = cos\,x - 1$	0	-2	1	1
9	$y = cos\,2x + 1$	2	0	1	2
10	$y = \|cos\,2x\| + 1$	2	1	1	2
11	$y = \|cos\,2x\| - 1$	0	-1	1	2

TEACHER'S NOTE

DISCUSSIONS

For the function $y = c + a \cos bx$, answer the questions

1. *Describe the relationship between **coefficients a** and the **shape** of the graph.*

   The coefficient, a gives the value of the amplitude or range of the graph.

2. *Describe how the difference in **coefficient c** will change the graphs' features*

   *Changing **c** will move the graph up or down by $|c|$ units.*

Graph	*Move up/down*	*No. of unit the graph move*
$y = \cos x$	*none*	*none*
$y = \cos x + 1$	*Move Up*	*1 unit*
$y = \cos x - 1$	*Move Down*	*1 unit*

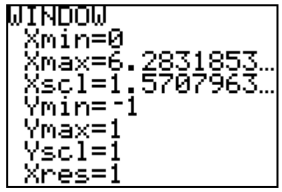
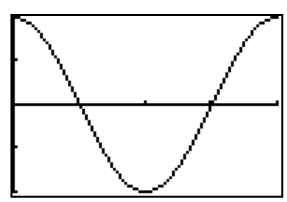
3. *Describe the relationship between **coefficients b** and the **shape** of the graph.*

   *The coefficient **b** gives the number of cycles in one rotation (360°=2π)*

   *Changing **b** affects the period. The period is $\frac{2\pi}{b}$ and **b** cannot be 0*

STUDENT'S WORKSHEET

Draw and sketch graph of given functions for $0 \le x \le 360°$ by using graphing calculator

No	Function	WINDOW setting	Graph
1	$y = cos\, 2x$		
2	$y = \lvert cos\, 2x \rvert$		
3	$y = -\lvert cos\, 2x \rvert$		
4	$y = cos \frac{1}{2} x$		
5	$y = 2\, cos\, x$		

6	$y = 3\cos 2x$		
7	$y = \cos x + 1$		
8	$y = \cos x - 1$		
9	$y = \cos 2x + 1$		
10	$y = \lvert\cos 2x\rvert + 1$		
11	$y = \lvert\cos 2x\rvert - 1$		

STUDENT'S WORKSHEET

INVESTIGATION

No	Graph	Maximum	Minimum	Amplitude	No of Cycle
1	$y = \cos 2x$	1			2
2	$y = \lvert\cos 2x\rvert$			1	
3	$y = -\lvert\cos 2x\rvert$	0			
4	$y = \cos\frac{1}{2}x$		-1		
5	$y = 2\cos x$				
6	$y = 3\cos 2x$				
7	$y = \cos x + 1$				
8	$y = \cos x - 1$				
9	$y = \cos 2x + 1$				
10	$y = \lvert\cos 2x\rvert + 1$				
11	$y = \lvert\cos 2x\rvert - 1$				

STUDENT'S WORKSHEET

DISCUSSIONS

For the function $y = c + a \cos bx$, answer the questions

1. *Describe the relationship between **coefficients a** and the **shape** of the graph.*

   The coefficient, a gives the value of the ……………………. or ……………………. of the graph.

2. *Describe how the difference in **coefficient c** will change the graphs' features*

   Changing c will move the graph …………… or ……………….. by ………………………..

Graph	*Move up/down*	*No. of unit the graph move*
$y = \cos x$	*none*	*none*
$y = \cos x + 1$		
$y = \cos x - 1$		

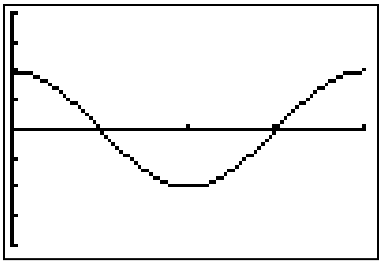
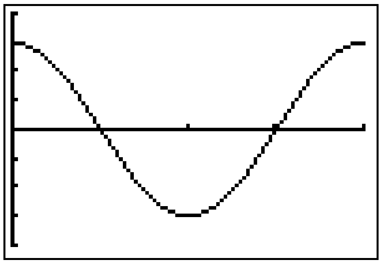
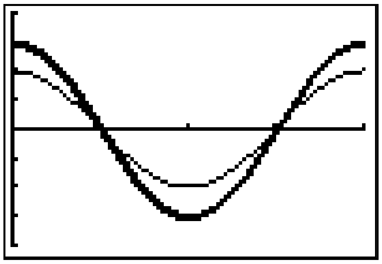
3. *Describe the relationship between **coefficients b** and the **shape** of the graph.*

   The coefficient b gives the ……………………………… in ………………………… (360°=2π)

   Changing b affects the ……………………. The period is …………………… and b cannot be 0

TEACHER'S NOTE

Draw the graph $y = c + a \cos bx$ below for $0° \leq x \leq 360°$ and answer the question

No	Function / Investigation	Graph / answer
1.	*y = 2 cos x*	
2.	*y = 3 cos x*	
3.	*Draw the two graphs above on the same axis*	
4.	*Write the conclusion*	*The coeficient **a** gives the amplitude of the graph.*

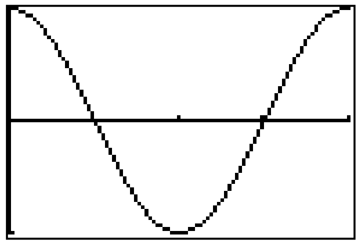
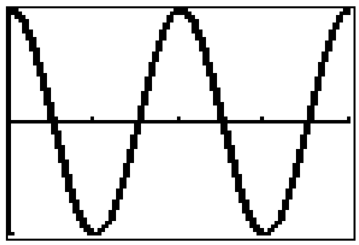
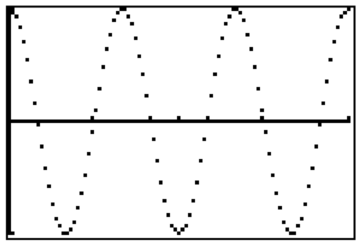
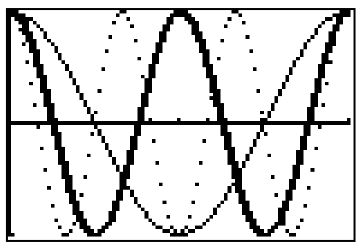
STUDENT'S WORKSHEET

Draw the graph $y = c + a\cos bx$ below for $0° \le x \le 360°$ and answer the question

No	Function / Investigation	Graph / Answer
1.	$y = 2\ \cos x$	
2.	$y = 3\ \cos x$	
3.	*Draw the two graphs above on the same axis*	
4.	*Write the conclusion*	

TEACHER'S NOTE

Draw the graph $y = c + a\cos bx$ below for $0° \leq x \leq 360°$ and answer the question

No	Function / Investigation	Graph / answer
1.	$y = \cos x$	
2.	$y = \cos 2x$	
3.	$y = \cos 3x$	
4.	*Draw the graphs above on the same axis*	
5.	*Write the conclusion*	*The coeficient **b** gives the number of cycle in one rotation of the graph.*

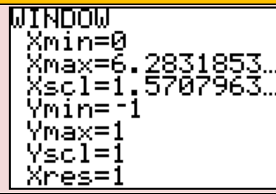
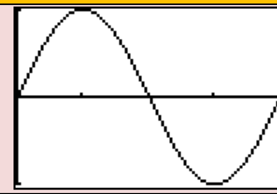
STUDENT'S WORKSHEET

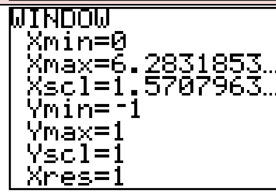
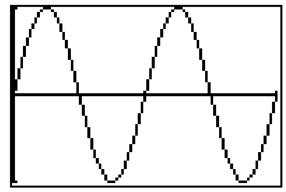
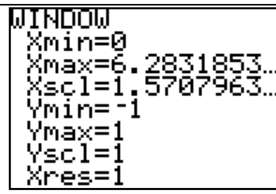
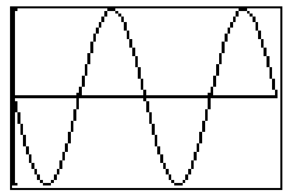
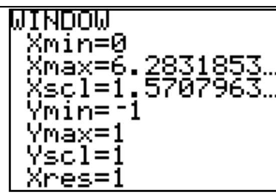
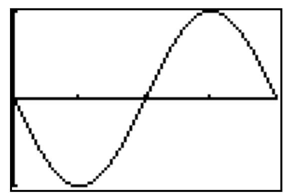
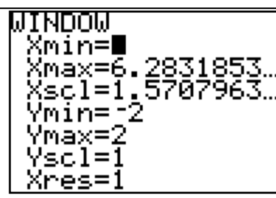
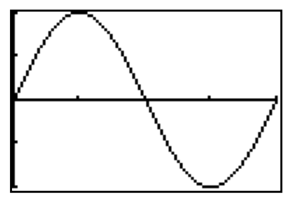
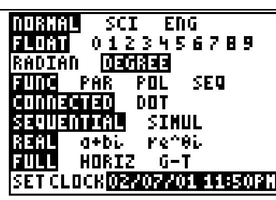
Draw the graph $y = c + a\cos bx$ below for $0° \leq x \leq 360°$ and answer the question

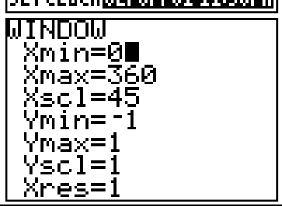
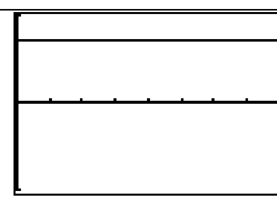
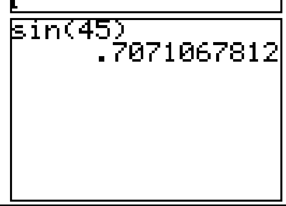
No	Function / Investigation	Graph / answer
1.	$y = \cos x$	
2.	$y = \cos 2x$	
3.	$y = \cos 3x$	
4.	*Draw the graphs above on the same axis*	
5.	*Write the conclusion*	

TEACHER'S NOTE

Draw the graph $y = c + a\sin bx$ below for $0° \leq x \leq 360°$ and answer the question

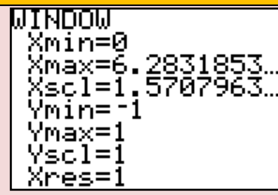
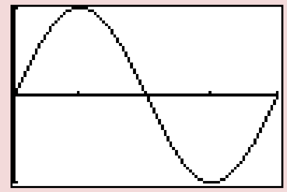
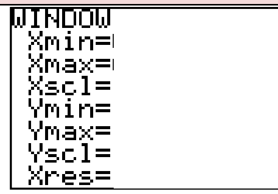
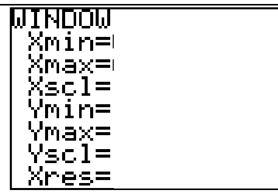
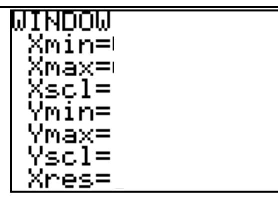
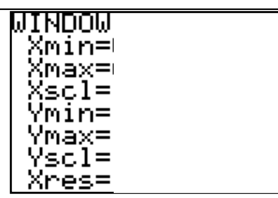
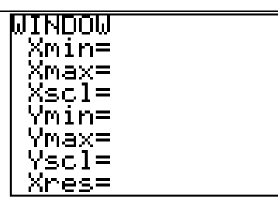
<table>
<tr><th>No</th><th>Function / Investigation</th><th>Window Setting</th><th>Graph / answer</th></tr>
<tr><td>Eg.</td><td>$y = \sin x$</td><td>
</td><td></td></tr>
<tr><td>1</td><td>y = sin 2x

Y= SIN 2 X,T,Θ,n)
GRAPH</td><td>
</td><td></td></tr>
<tr><td>2</td><td>y = - sin 2x</td><td>
</td><td></td></tr>
<tr><td>3</td><td>y = - sin x</td><td>
</td><td></td></tr>
<tr><td>4</td><td>y = 2 sin x</td><td>
</td><td></td></tr>
<tr><td>5</td><td>$y = \sin 45^\circ$</td><td>


</td><td>

</td></tr>
</table>

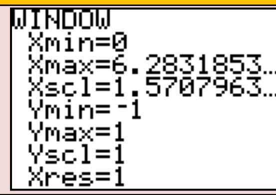
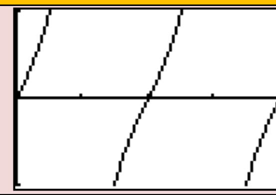
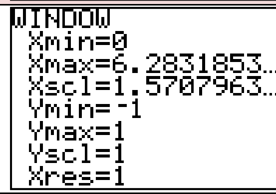
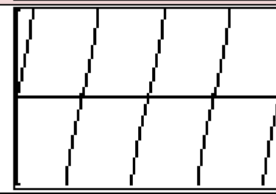
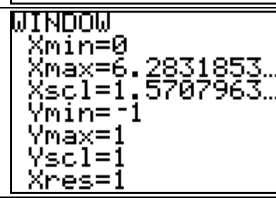
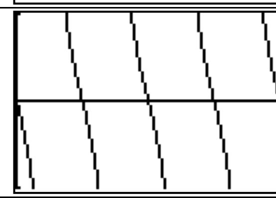
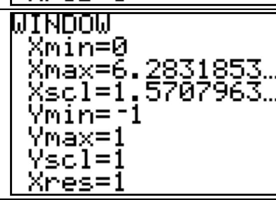
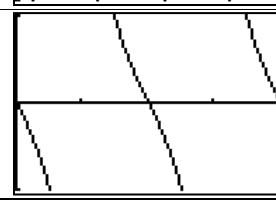
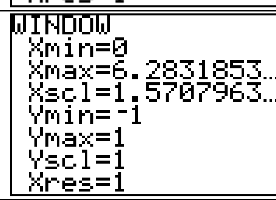
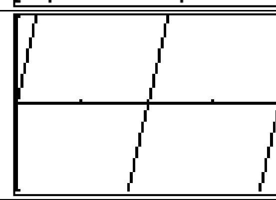
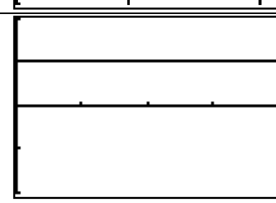
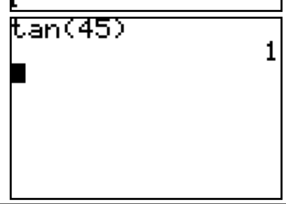
STUDENT'S WORKSHEET

Draw the graph $y = c + a \sin bx$ below for $0° \leq x \leq 360°$ and answer the question

No	Function / Investigation	Window Setting	Graph / answer
Eg.	$y = \sin x$		
1	*y = sin 2x* $\boxed{Y=}$ $\boxed{SIN}$ $\boxed{2}$ $\boxed{X,T,\Theta,n}$ $\boxed{)}$ $\boxed{GRAPH}$		
2	*y = - sin 2x*		
3	*y = - sin x*		
4	*y = 2 sin x*		
5	$y = \sin 45^{\circ}$		

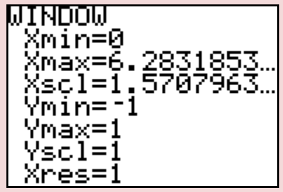
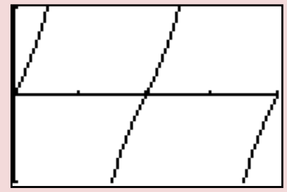
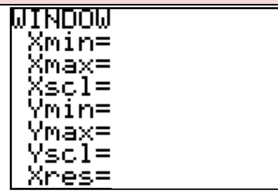
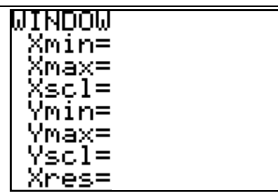
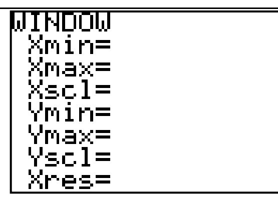
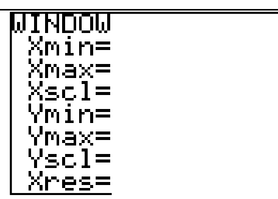
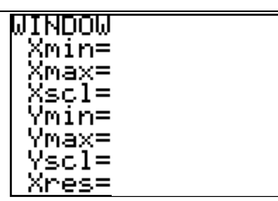
TEACHER'S NOTE

Draw the graph $y = c + a \tan bx$ below for $0° \leq x \leq 360°$ and answer the question

No	Function / Investigation	Window Setting	Graph / answer
Eg.	$y = \tan x$		
1	*y = tan 2x* Y= TAN 2 X,T,Θ,n GRAPH		
2	*y = – tan 2x*		
3	*y = – tan x*		
4	*y = 2 tan x*		
5	$y = \tan 45°$	 	 

STUDENT'S WORKSHEET

Draw the graph $y = c + a\tan bx$ below for $0° \leq x \leq 360°$ and answer the question

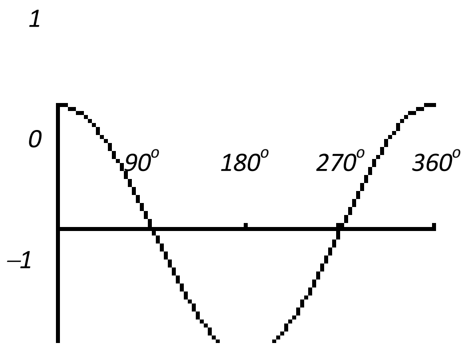
No	Function / Investigation	Window Setting	Graph / answer
Eg.	$y = \tan x$		
1	*y = tan 2x* [Y=] [TAN] [2] [X,T,Θ,n] [GRAPH]		
2	*y = – tan* $2x$		
3	*y = – tan* x		
4	*y = 2 tan x*		
5	$y = \tan 45°$		

STUDENT'S WORKSHEET

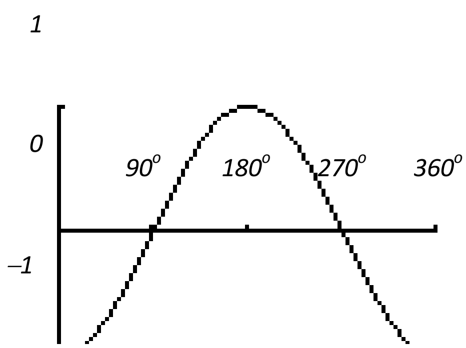
SPM QUESTIONS

1. *Which of the following graph represent* $y = \cos 2x$ *?*

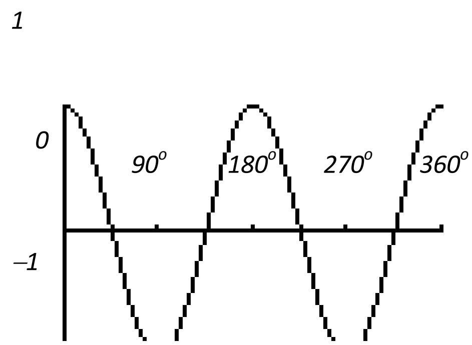
A



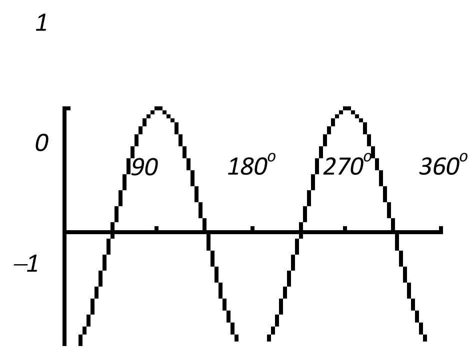
B



C



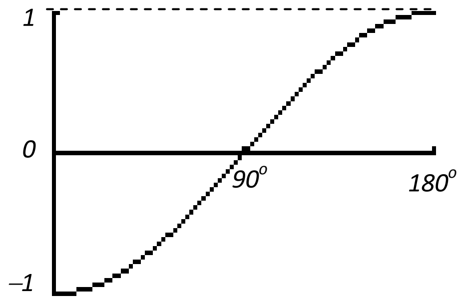
D



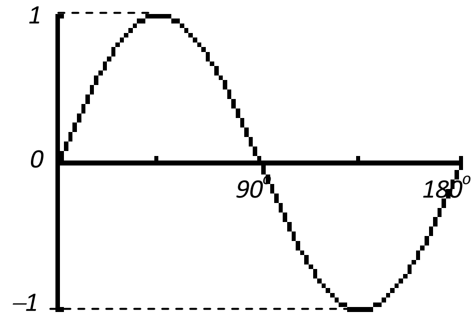
SPM 2005

2. *Which of the following represents the graph of* $y = \cos x$ *for* $0^\circ \leq x \leq 180^\circ$?

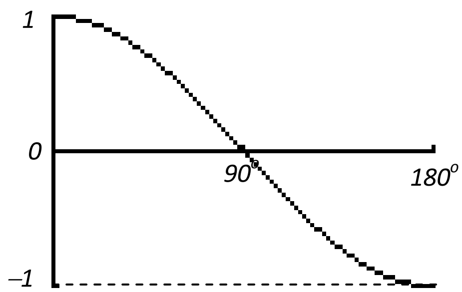
A



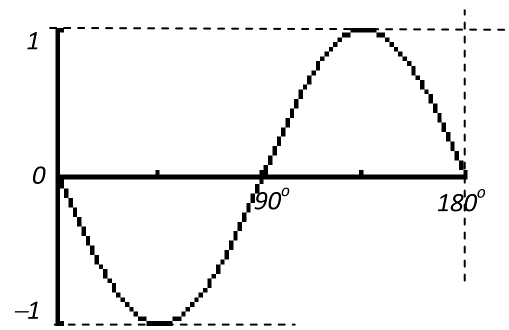
C



B



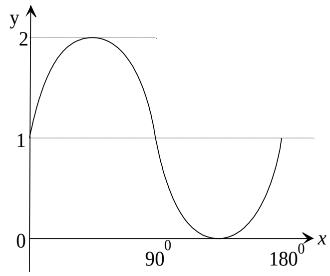
D



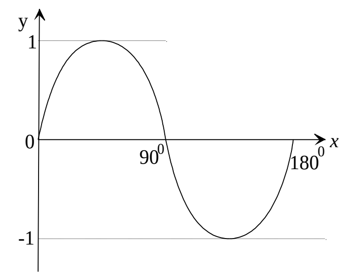
SPM 2006

3. *Which of the following graphs represents y = sin 2x for 0° ≤ x° ≤ 180?*

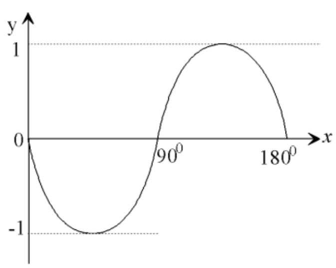
A



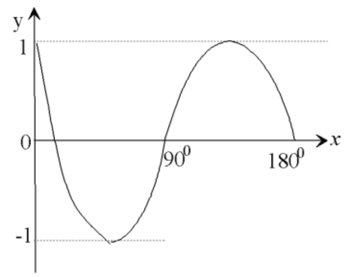
C



B

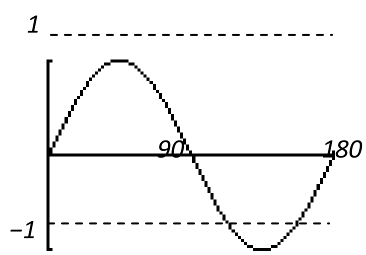


D



4 *Which of the following graphs represents* $y = \sin x$ for $0^\circ \le x \le 180^\circ$ ***?***

A



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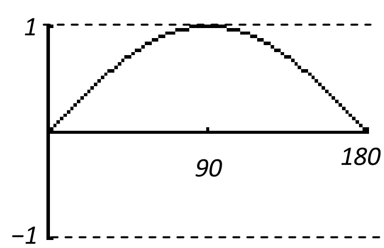
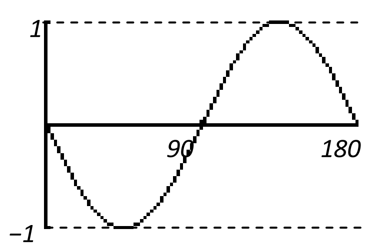
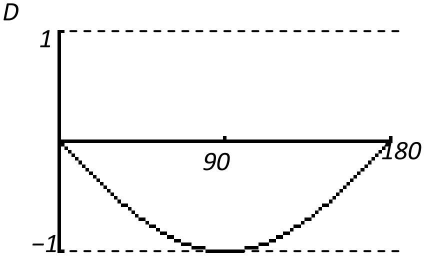


B



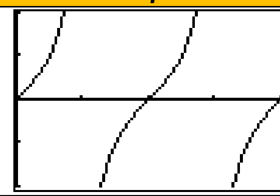
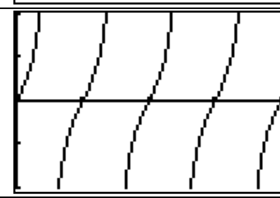
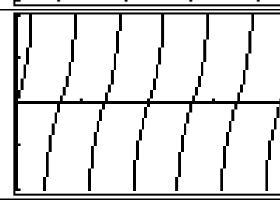
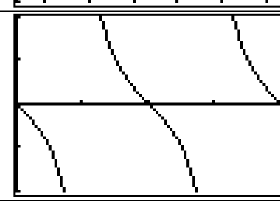
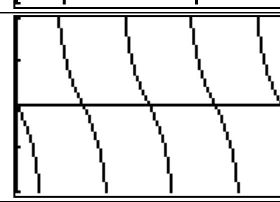
D



TEACHER'S NOTE

Graphs of Tangent

Predict and sketch the graphs of the following tangent functions for $0 \leq x \leq 2\pi$.
Test your predictions by drawing the tangent graph by using graphic calculator.

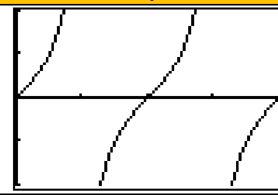
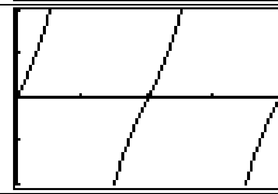
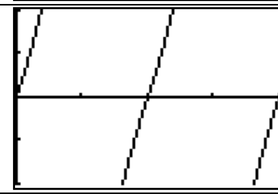
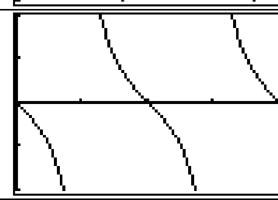
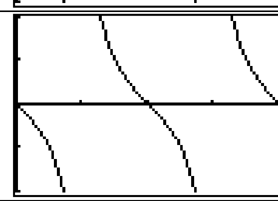
No.	***a***	***b***	***c***	$y = a\ \tan bx + c$	**Graphs**
1	*1*	*1*	*0*	$y = \tan x$	
2	*1*	*2*	*0*	$y = \tan 2x$	
3	*1*	*3*	*0*	$y = \tan 3x$	
4	*1*	*-1*	*0*	$y = \tan(-x)$	
5	*1*	*-2*	*0*	$y = \tan -2x$	

Conclusion:

*As the value of **b** **increase**, the number of complete cycle **increases** accordingly*

*As the value of **b** **decrease**, the number of complete cycle **decreases** accordingly*

TEACHER'S NOTE

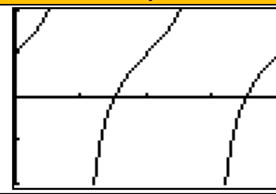
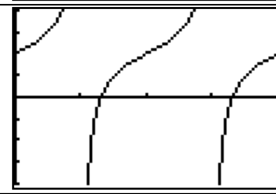
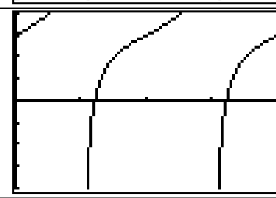
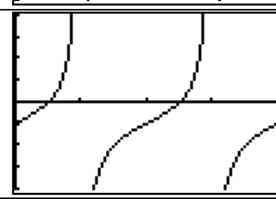
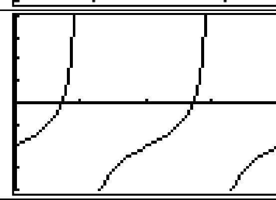
No.	a	b	c	$y = a\ \tan bx + c$	Graphs
6	1	1	0	$y = \tan x$	
7	2	1	0	$y = 2\tan x$	
8	3	1	0	$y = 3\tan x$	
9	-1	1	0	$y = -\tan x$	
10	-2	1	0	$y = -2\tan x$	

Conclusion:

*As the value of **a** **increase,** the graph becomes **narrower***

*As the value of **a** **decrease**, the graph becomes **wider***

TEACHER'S NOTE

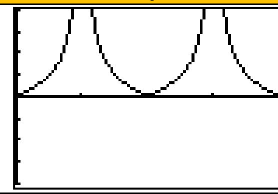
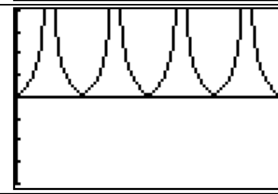
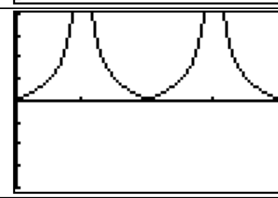
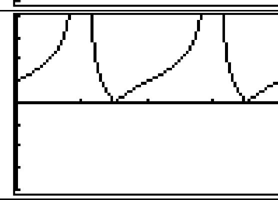
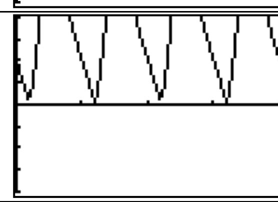
No.	**a**	**b**	**c**	$\boldsymbol{y = \tan x + 1}$	**Graphs**
11	*1*	*1*	*1*	$y = \tan x + 2$	
12	*1*	*1*	*2*	$y = \tan x + 3$	
13	*1*	*1*	*3*	$y = \tan x - 1$	
14	*1*	*1*	*-1*	$y = \tan x - 2$	
15	*1*	*1*	*-2*	$y = \tan x - 2$	

Conclusion:

*As the value of **c** **increase,** the graph is shifted **upwards.***

*As the value of **c** **decrease**, the graph is shifted **downwards***

TEACHER'S NOTE

No.	**a**	**b**	**c**	$y = \lvert \tan x \rvert$	**Graphs**
16	*1*	*1*	*0*	$y = \lvert \tan 2x \rvert$	
17	*1*	*2*	*0*	$y = \lvert -\tan x \rvert$	
18	*-1*	*1*	*0*	$y = \lvert \tan x + 1 \rvert$	
19	*1*	*1*	*1*	$y = \lvert 3\tan 2x - 2 \rvert$	
20	*3*	*2*	*-2*	$y = \lvert 3\tan 2x - 2 \rvert$	

Conclusion:

*The **absolute** value of a trigonometric function causes **y** value to be **positive** due to being reflected **upwards** about the x-axis*

STUDENT'S WORKSHEET

Graphs of Tangent

Predict and sketch the graphs of the following tangent functions for $0 \le x \le 2\pi$.
Test your predictions by drawing the tangent graph by using graphic calculator.

No.	a	b	c	$y = a\ \tan bx + c$	Graphs
1	1	1	0	$y = \tan x$	
2	1	2	0	$y = \tan 2x$	
3	1	3	0	$y = \tan 3x$	
4	1	-1	0	$y = \tan(-x)$	
5	1	-2	0	$y = \tan -2x$	

Conclusion:

As the value of b ……………, the number of complete cycle …………… accordingly

As the value of b …………… the number of complete cycle ……………. accordingly

STUDENT'S WORKSHEET

No.	a	b	c	$y = a \tan bx + c$	Graphs
6	1	1	0	$y = \tan x$	
7	2	1	0	$y = 2\tan x$	
8	3	1	0	$y = 3\tan x$	
9	-1	1	0	$y = -\tan x$	
10	-2	1	0	$y = -2\tan x$	

Conclusion:

As the value of a……………, the graph becomes …………………..

As the value of a ……………., the graph becomes …………………

STUDENT'S WORKSHEET

No.	a	b	c	$y = \tan x + 1$	Graphs
11	1	1	1	$y = \tan x + 2$	
12	1	1	2	$y = \tan x + 3$	
13	1	1	3	$y = \tan x - 1$	
14	1	1	-1	$y = \tan x - 2$	
15	1	1	-2	$y = \tan x - 2$	

Conclusion:

As the value of c ………………., the graph is shifted ……………….

As the value of c ………………., the graph is shifted ……………….

STUDENT'S WORKSHEET

No.	**a**	**b**	**c**	$y = \lvert \tan x \rvert$	**Graphs**
16	1	1	0	$y = \lvert \tan 2x \rvert$	
17	1	2	0	$y = \lvert -\tan x \rvert$	
18	-1	1	0	$y = \lvert \tan x + 1 \rvert$	
19	1	1	1	$y = \lvert 3\tan 2x - 2 \rvert$	
20	3	2	-2	$y = \lvert 3\tan 2x - 2 \rvert$	

Conclusion:

The …………. value of a trigonometric function causes y value to be ……………. due to being reflected …………… about the x-axis

TEACHER'S NOTE

INVESTIGATION

1. *The effect of a, b and c in the trigonometric functions* $y = a\tan(bx) + c$ *is*

   *As the value of **a** **increase**, the graph becomes **narrower***

   *As the value of **a** **decrease**, the graph becomes **wider***

   *As the value of **b** **increase**, the number of complete cycle **increases** accordingly*

   *As the value of **b** **decrease**, the number of complete cycle **decreases** accordingly*

   *As the value of **c** **increase**, the graph is shifted **upwards.***

   *As the value of **c** **decrease**, the graph is shifted **downwards***

2. *The absolute value of a trigonometric function causes its graph to*

   *The absolute value of a trigonometric function causes **y** value to be **positive** due to being reflected **upwards** about the x-axis*

STUDENT'S WORKSHEET

INVESTIGATION

1. *The effect of a, b and c in the trigonometric functions* $y = a\tan(bx) + c$ *is*

   As the value of ***a*** *………………., the graph becomes ……………………….*

   As the value of ***a*** *………………., the graph becomes ………………………*

   As the value of ***b*** *……………., the number of complete cycle ……………. accordingly*

   As the value of ***b*** *……………., the number of complete cycle ……………. accordingly*

   As the value of ***c*** *……………., the graph is shifted ……………..*

   As the value of ***c*** *……………., the graph is shifted ……………..*

2. *The absolute value of a trigonometric function causes its graph to*

   The absolute value of a trigonometric function causes ***y*** *value to be …………… due to being reflected*

   ……………. about the x-axis

Topic **:** ***TRIGONOMETRY II***

Learning Objective **:**

1. *Understand and use the concept of the values of* $\sin\theta$, $\cos\theta$ *and* $\tan\theta$ *for* $0° \le \theta \le 360°$ *to solve problems.*
2. *Draw and use the graphs of sine, cosine and tangent.*

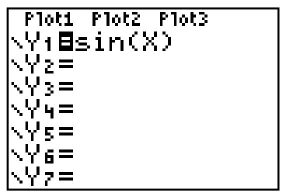
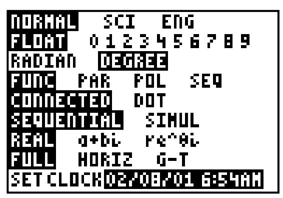
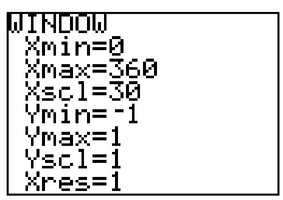
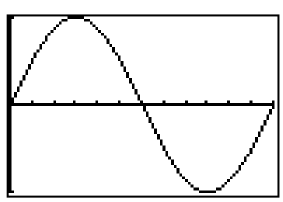
Lesson Objective **:**

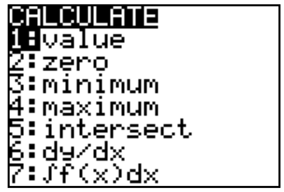
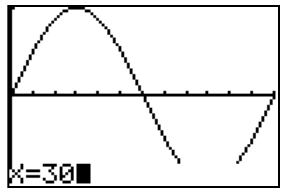
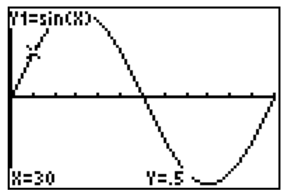
Students will be able to…

i. *Find the values of sine, cosine and tangent for angles between* 0° *and* 360°.

EXAMPLE QUESTION **:** ***Find the values of*** $\sin\theta$ ***for*** $0° \le \theta \le 360°$ ***and fill in the table below***

x	*0°*	*30°*	*60°*	*90°*	*120°*	*150°*	*180°*	*210°*	*240°*	*270°*	*300°*	*330°*	*360°*
y													

Step	***Procedure***	***Screenshot / key-stroke***	***Notes***
1	*Key in the function* *y = sin x.*		
2	*Press* [MODE] *Setup for* ***DEGREE*** *press* [ENTER]		
3	*Setup the* [WINDOW]		• *X-axis scale is 30° for 1 unit.*
4	*Press* [GRAPH]		

5	*Press* [2nd] [TRACE], *on* ***CALCULATE*** *menu, choose* ***1:value****,*		
6	*Key in the value for x* ***X=30*** *Press* [ENTER] *Repeat step 5 for all the x- values*	 	*When x = 30°, then y=0.5*

The values of $\boldsymbol{sin\,\theta}$ ***for*** $\mathbf{0° \leq \theta \leq 360°}$

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
y	0	0.5	0.86	1	0.86	0.5	0	-0.5	-	-1	-0.866	-0.5	0

TEACHER'S NOTE

1. ***Find the values of $\underline{\cos\theta}$ for $0° \leq \theta \leq 360°$ and fill in the table below***

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
y	1	0.866	0.5	0	-0.5	-	-1	-	-0.5	0	0.5	0.866	1

2. ***Find the values of $\underline{\tan\theta}$ for $0° \leq \theta \leq 360°$ and fill in the table below***

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
y	0	0.577	1.732	UnDef	-	-	0	0.577	1.732	UnDef	-	-	0

STUDENT'S WORKSHEET

1. ***Find the values of $\cos\theta$ for $0^\circ \leq \theta \leq 360^\circ$ and fill in the table below***

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
y													

2. ***Find the values of $\tan\theta$ for $0^\circ \leq \theta \leq 360^\circ$ and fill in the table below***

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
y													

TOPIC : ***TRIGONOMETRIC FUNCTIONS***

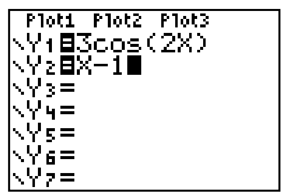
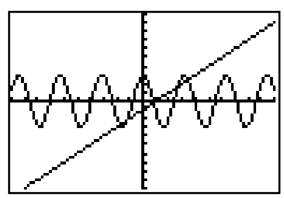
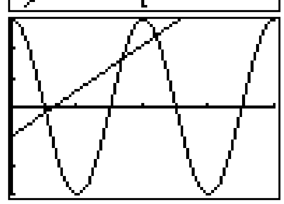
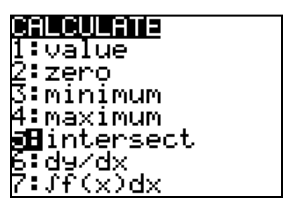
SUB TOPIC : *GRAPHS OF SINE, COSINE AND TANGENT FUNCTIONS.*

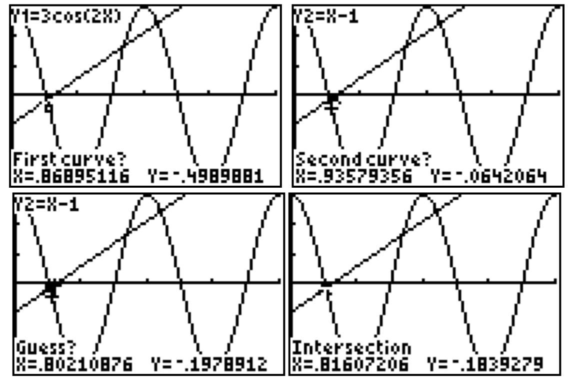
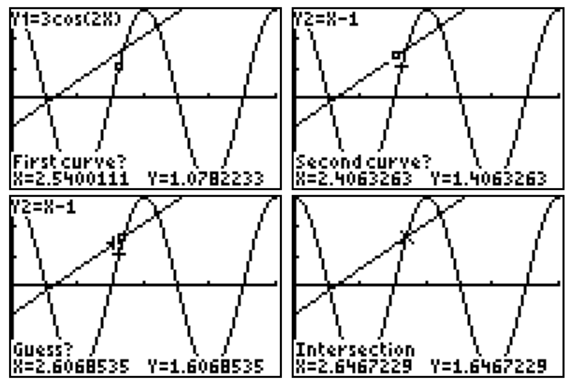
LESSON OBJECTIVES :

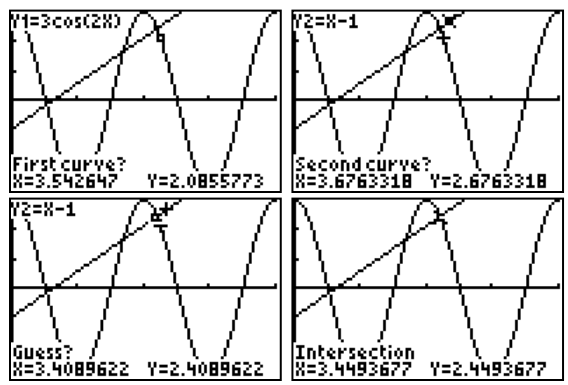
Students will be able to…

i. *Determine the number of solutions to a trigonometric equations using sketched graph.*
ii. *Solve trigonometric equations using drawn graphs.*

EXAMPLE QUESTION : *On the same axes, sketch the graphs* $y = 3\ \cos 2x$ *and* $y = x - 1$ *for* $0 \leq x \leq 2\pi$. *Hence, find the* ***number of solutions*** *and the values of* x *for the equation* $3\ \cos 2x + 1 = x$.

Step	***Procedure***	***Screenshot / key-stroke***	***Notes***
1	*Key in the functions* $y = 3\ \cos 2x$ ***and*** $y = x - 1$		
2	*Press* $\boxed{\text{GRAPH}}$ *(Adjust the Window Setting accordingly)*	 	*Can you see the intersection between the two graphs* *How many intersection points?* ***3 intersection points means 3 solutions***
3	***TO FIND THE INTERSECTION*** *Press* $\boxed{\text{2nd}}\boxed{\text{TRACE}}$ *for* ***CALC*** *mode* *Choose* ***5:intersect***		

<table>
<tr><td></td><td></td><td>
</td></tr>
<tr><td>4</td><td>Repeat the same procedure for another two intersection points.</td><td>


</td></tr>
<tr><td>5</td><td colspan="2">

FINALIZE THE ANSWER:

Intersection	***x***	*Y*	*coordinate*
A	***0.82***	*-0.18*	$(0.82, -0.18)$
B	***2.65***	*1.65*	$(2.65, 1.65)$
C	***3.45***	*2.45*	$(3.45, 2.45)$

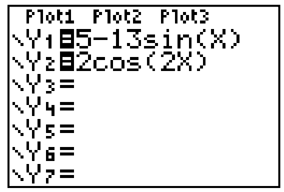
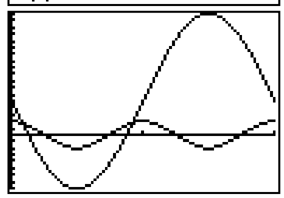
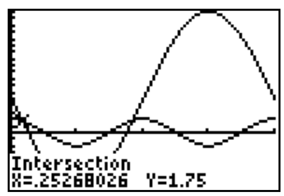
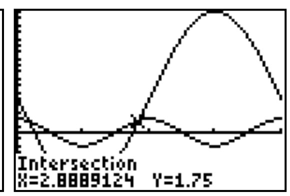
∴ 3 Solutions

</td></tr>
</table>

TEACHER'S NOTE

Solve the equation $\mathbf{5 - 13\ sin\,x = 2\ \ cos\,2x}$ *for* $\mathbf{0° \le x \le 360°}$.
Hence, find the ***number of solutions***

ANSWER:

Step	***Procedure***	***Screenshot / key-stroke***	***Notes***
1	*Key in the functions* $\mathbf{y = 5 - 13\ \ sin\,x}$ *and* $\mathbf{y = 2\ \ cos\,2x}$		***(the setting is in radian)*** *From the graph, it has* ***2 intersections, which means*** ***2 solutions.***
2	*Find the intersection points*	 *Therefore,* *The intersection points are* $\mathbf{(0.25, 1.75)}$***, and*** $\mathbf{(2.89, 1.75)}$	

STUDENT'S WORKSHEET

Solve the equation $\mathbf{5-13\ \sin x=2\ \cos 2x}$ *for* $\mathbf{0°\le x\le 360°}$.
Hence, find the ***number of solutions***

ANSWER:

Step	***Procedure***	***Screenshot / key-stroke***
1	*Key in the functions*	
2	*Find the intersection points*	
3	*Finalize the answer*	

TOPIC : ***LINEAR PROGRAMMING***

SUBTOPIC : THE CONCEPT OF LINEAR INEQUALITIES

LESSON OBJECTIVE :

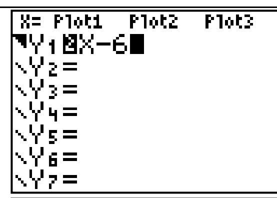
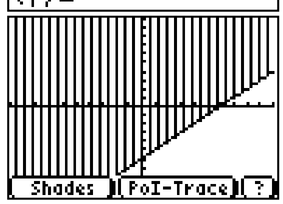
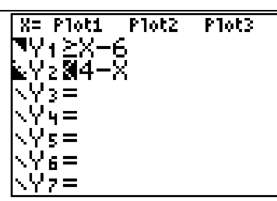
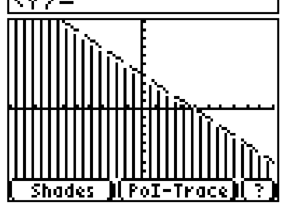
Students will be able to…

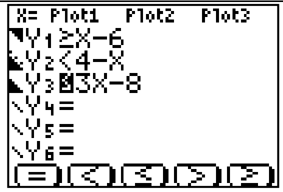
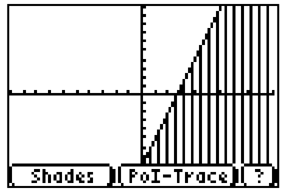
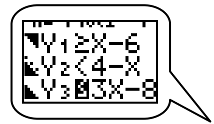
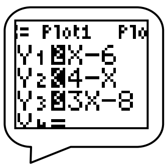
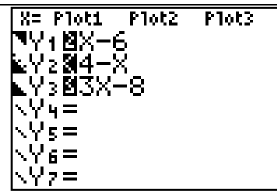
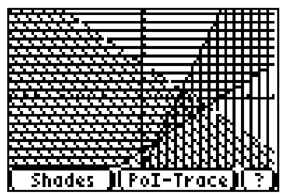
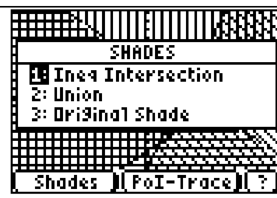
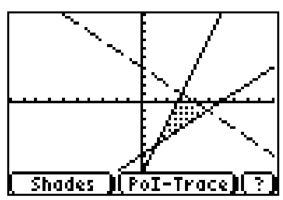
i. Identify and shade the region in which every point satisfies a linear inequality

ii. Find the linear inequality that defines a shaded region

EXAMPLE QUESTION : *For each of the following, identify and shade the region in which every point satisfies the given linear inequality:*

$$y \geq x-6$$
$$2x+2y<8$$
$$y \leq 3x-8$$

Step	***Procedure***	***Screenshot / key-stroke***	***Notes***
1	Press $\boxed{\text{APPS}}$; Scroll down until find **:Inequalz**. $\boxed{\textit{ENTER}}$		
2	Key in the equation $y \geq x-6$ *Press* $\boxed{\textit{ALPHA}}\boxed{\textit{GRAPH}}$ *to set the inequalities function.* Press $\boxed{\text{GRAPH}}$		
3	Key in the equation $2x+2y<8$		*Make sure to* ***unhighlight*** *the first function.* *Press* $\boxed{\textit{ENTER}}$ *on the inequalities symbol* $2x+2y<8$ $2y<8-2x$ $y<4-x$

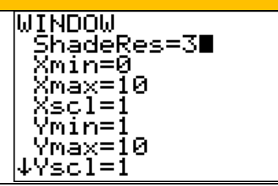
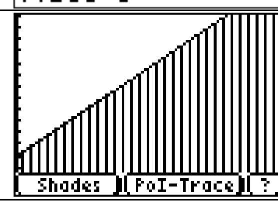
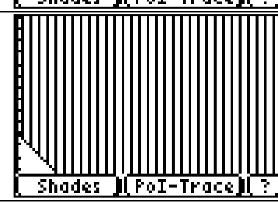
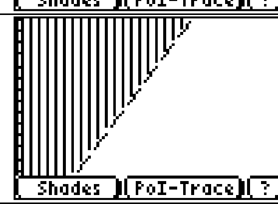
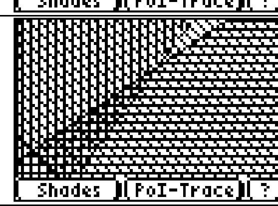
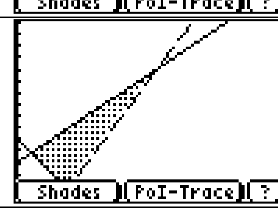
4	Key in the equation $y \leq 3x - 8$	 	 *Make sure to **unhighlight** the first and second function*
5	 **Highlight** all the functions	 	
6	[ALPHA] [Y=] for **shades** mode Choose **1: Ineq Intersection**	 	

TEACHER'S NOTE

For each of the following, identify and shade the region in which every point satisfies the given linear inequality, for $x > 0$, and $y > 0$

$$y \leq x + 3$$
$$2y - 10 \geq -\frac{3}{2}x$$
$$y > 2x - 5$$

SOLUTIONS:

Step	***Procedure***	***Screenshot / key-stroke***
1	*Suggested window setting*	
2	$y \leq x + 3$	
3	$2y - 10 \geq -\frac{3}{2}x$ *Change y as the subject*	
4	$y > 2x - 5$	
5	*Highlight all the functions*	
6	ALPHA Y= for **shades** mode *Choose **1: Ineq Intersection***	

STUDENT'S WORKSHEET

For each of the following, identify and shade the region in which every point satisfies the given linear inequality, for $x > 0$, and $y > 0$

$$y \leq x + 3$$
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$$y > 2x - 5$$

SOLUTIONS:

Step	***Procedure***	***Screenshot / key-stroke***
1	$y \leq x + 3$	
2	$2y - 10 \geq -\frac{3}{2}x$	
3	$y > 2x - 5$	
4	*Highlight all the functions*	
5	ALPHA Y= for **shades** mode *Choose **1: Ineq Intersection***	

TOPIC : ***LINEAR PROGRAMMING***

SUBTOPIC : THE CONCEPT OF LINEAR PROGRAMMING

LESSON OBJECTIVE :

Students will be able to…

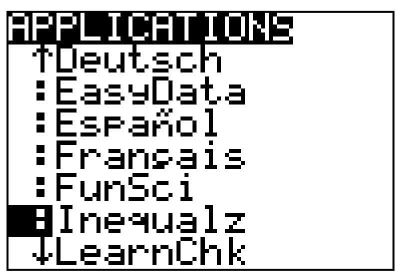
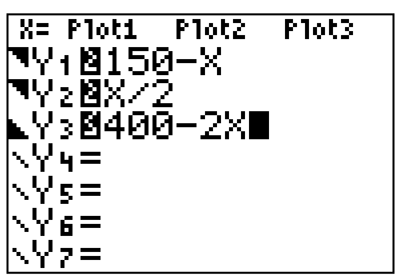
i. Solve problems related to linear programming by shading the region where the points in the region are feasible solutions.

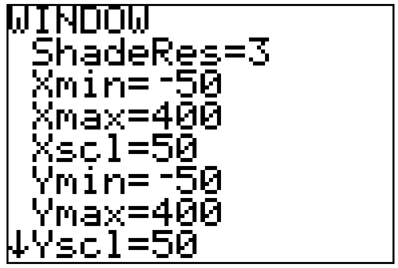
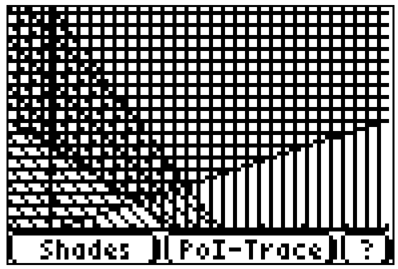
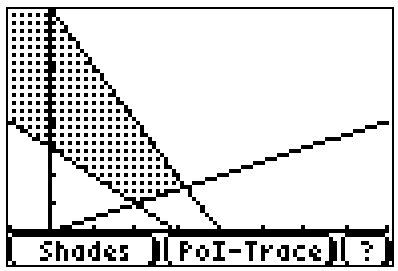
EXAMPLE QUESTION : *Mukhriz intends to sell two types of computer printers A and B. He buys the printers from the computer company with the following conditions:*

I : the total number of printers is at least 150.
II : the number of printer B is at least half the number of printer A.
III : Printer A costs RM400 and printer B costs RM200. Mukhriz is able to invest a maximum of RM80000.

(a) If x and y represent the number of printers A and B respectively, write down three inequalities other than $x \geq 0$ and $y \geq 0$ which satisfy the above conditions.
[3 marks]

(b) By using a scale of 2 cm to 50 units on both axes, construct and shade the region R which satisfies all the above conditions.
[3 marks]

Step	***Procedure***	***Screenshot / key-stroke***	***Notes***
1	Press [APPS]-key; Scroll down until find **:Inequalz**. *[ENTER]*		
2	Key in the equations: $x+y \geq 150$ $y \geq \frac{x}{2}$ $2x+y \leq 400$		$x+y \geq 150$ $y \geq 150-x$ $y \geq \frac{x}{2}$ $2x+y \leq 400$ $y \leq 400-2x$

		WINDOW ShadeRes=3 Xmin=-50 Xmax=400 Xscl=50 Ymin=-50 Ymax=400 ↓Yscl=50 Shades PoI-Trace ? Shades PoI-Trace ?	

TEACHER'S NOTE

Activity 1

The Mathematics and Science department in a school is organizing a camp for students. The camp will be attended by x male and y female students. The selection of camp participants is based on the following conditions:

I	***:***	***The total number of participants is at least 30.***
II	***:***	***The number of male students exceeds the number of female students by a maximum of 20.***
III	***:***	***The expenditure per male students and per female students is RM20 and RM40 respectively and the maximum allocation for the camp is RM1600.***

(a) ***Find three linear inequalities other than*** $x \geq 0$ ***and*** $y \geq 0$ ***which satisfy the above conditions.***

[3 marks]

(b) ***By using a scale of 2cm to 10 participants on axis x and y, construct and shade the region R that satisfies all the above conditions.***

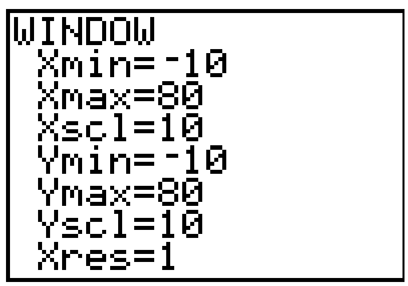
[3 marks]

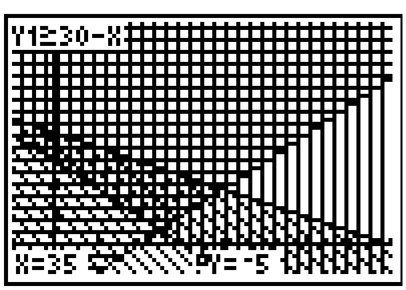
Solutions:

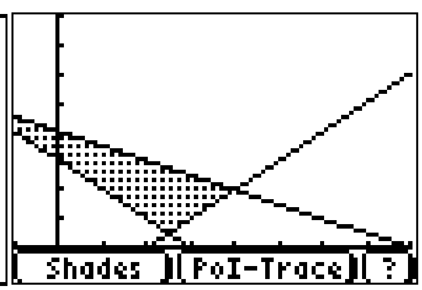
I : $x + y \geq 30$

II : $y \geq x - 20$

III : $x + 2y \leq 80$







STUDENT'S WORKSHEET

Activity 1

The Mathematics and Science department in a school is organizing a camp for students. The camp will be attended by x male and y female students. The selection of camp participants is based on the following conditions:

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[3 marks]

(b) ***By using a scale of 2cm to 10 participants on axis x and y, construct and shade the region R that satisfies all the above conditions.***

[3 marks]

Solutions:

TEACHER'S NOTE

ENRICHMENT

SPM 2005 (PAPER 2: SECTION C: NO.14)

An institution offers two computer courses, P and Q. The number of participants for courses P is x and for course Q is y.
The enrolment of the participants is based on the following constraints:

I : The total number of participants is not more than 100
II : The number of participants for course Q is not more than 4 times the number of participants for course P.
III : The number of participants for course Q must exceed the number of participants for course P by at least 5.

*a) Write down three inequalities, other than $x \geq 0$ **and** $y \geq 0$, which satisfy all the above constraints.*
[3 marks]

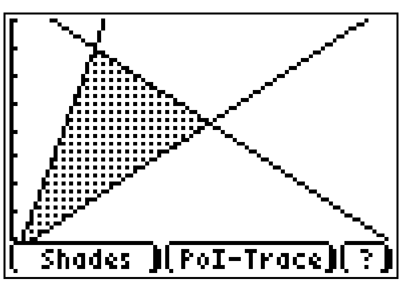
b) By using a scale of 2 cm to 10 participants on both axes, construct and shade the region R that satisfies all the above constraints.
[3 marks]

c) By using your graph from (b), find
(i) The range of the number of participants for course Q if the number of participants for course P is 30
(ii) The maximum total fees per month that can be collected if the fees per month for courses P and Q are RM50 and RM60 respectively.
[4 marks]

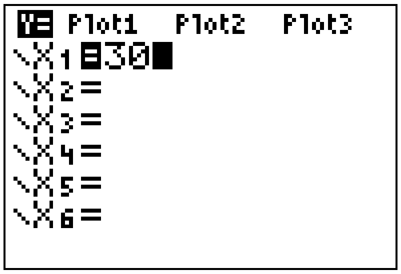
Answer :

a) I : $x + y \leq 100$
II : $y \leq 4x$
III : $y \geq x + 5$

b)

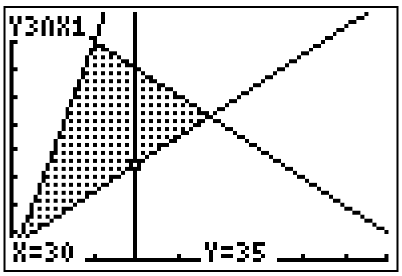


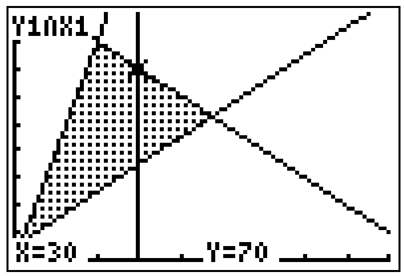
c) (i) Draw line x= 30



Press GRAPH

Press TRACE to read the intersection values





When $x = 30$, $35 \le y \le 70$

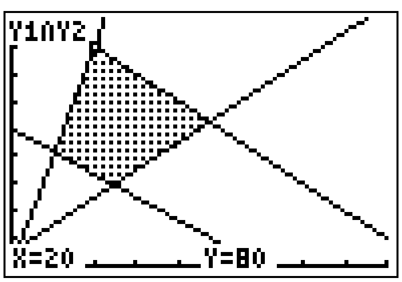
(ii) Total fees , $k = 50x + 60y$

If $k = 3000$, $3000 = 50x + 60y$

K maximum if $x = 20, y = 80$

Total fees maximum = 50(20) + 60(80)

= RM5800



STUDENT'S WORKSHEET

ENRICHMENT

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a) *Write down three inequalities, other than $x \geq 0$ and $y \geq 0$, which satisfy all the above constraints.*
[3 marks]
b) *By using a scale of 2 cm to 10 participants on both axes, construct and shade the region R that satisfies all the above constraints.*
[3 marks]
c) *By using your graph from (b), find*
   (i) *The range of the number of participants for course Q if the number of participants for course P is 30*
   (ii) *The maximum total fees per month that can be collected if the fees per month for courses P and Q are RM50 and RM60 respectively.*
[4 marks]

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